Noise Minimization in Quantum Transistors

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Very helpful discussions with Patrice Roche (Saclay), Christian Glattli (Saclay, ENS) and Yehoshua Levinson (Weizmann)

Modern Transistors (Single Electron Transistors, Molecular Transistors…) are expected to operate close to the fundamental limits that quantum mechanics imposes on their performances

Aim:

Given a transistor obtain experimentally the best possible signal-to-noise ratio for a “very quantum signal”

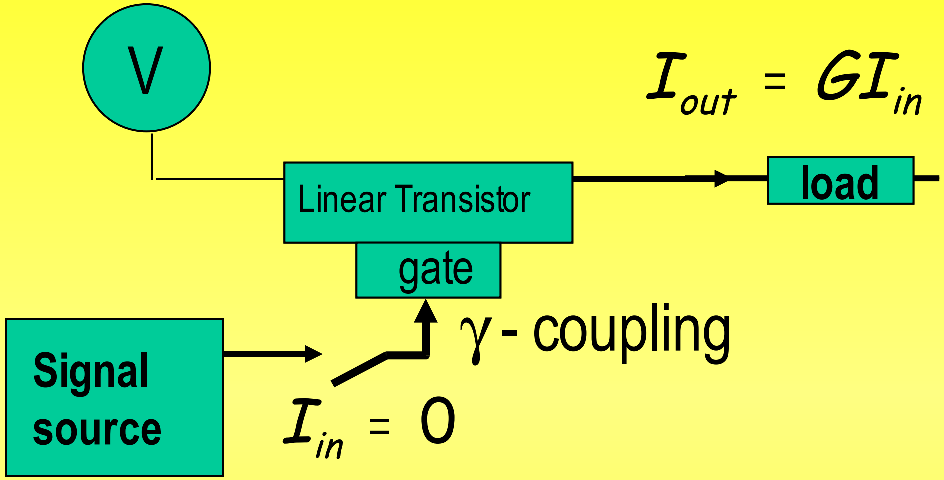
“very quantum signal” – quantum fluctuations are large compared to the expectation value
Example: the current in the ground state

“experimentally” – practical procedure for a given actual device, not a given Hamiltonian,
That is, a black-box noise-minimization procedure

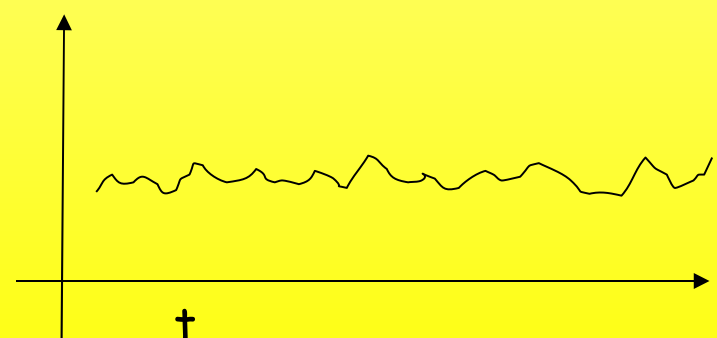
Idling noise and amplified back-action noise

$I_{out} = GI_{in} + I_{Noise}$

Disconnected signal source



$I_{Noise} = I_0$

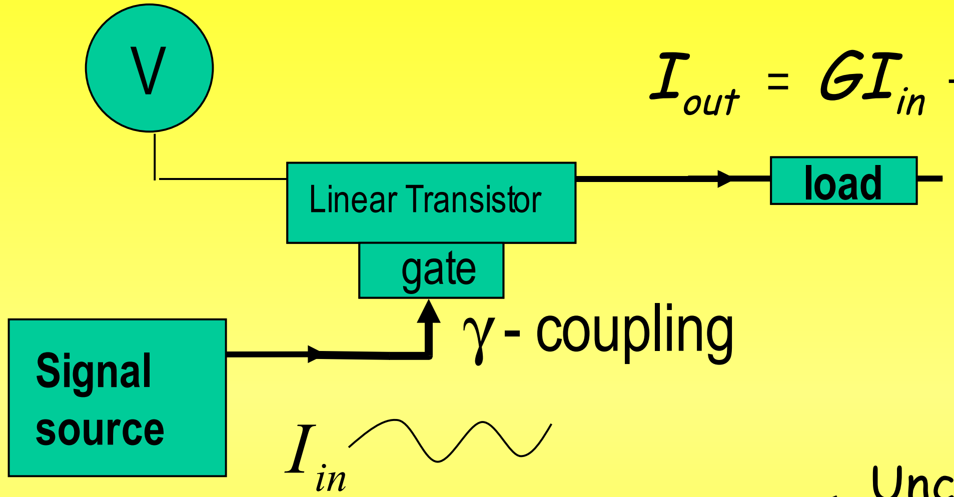


$\Delta I^2_{noise} \equiv \Delta I_o^2$ idling noise

thermal noise, zero-point noise, shot noise, 1/f noise, etc

Idling noise and amplified back-action noise

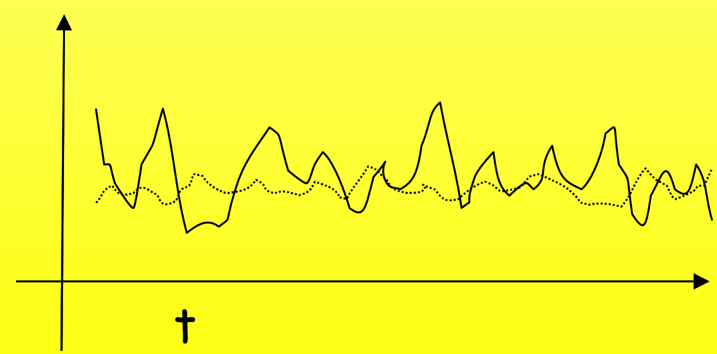
$$I_{out} = GI_{in} + I_{Noise}$$



Connected signal source

Uncorrelated

$$I_{Noise} = I_0 + I_n$$



$$\Delta I_{Noise}^2 \equiv \Delta I_o^2 + \Delta I_n^2$$

ΔI_n^2 amplified back-action noise

(feedback noise)

What is the minimal value of ΔI_{noise} ?

Ideal classical linear amplifiers: $\Delta I^2_{Noise} = 0$

Ideal quantum linear amplifiers:

$$\Delta I^2_{Noise} \geq (G^2 - 1)\frac{\hbar\omega}{2} g_s \Delta \nu$$

Caves, PRD 26, 1817, '82 (noconserved boson)

Gavish, Yurke, Imry PRL 95, 020401, '04 (General, using nonequilib Kubo's FDT)

In terms of the idling and back-action noise of transistors:

$$\Delta I_0 \Delta I_n \geq G^2 \Delta I^2_{zp} \overset{\text{impedance-matching}}{=} G^2 \frac{\hbar\omega}{2} g_s \Delta \nu$$

Gavish, Yurke, Imry PRL 96, 133602, '06

Related works: Averin, cond-mat/0301524,

Clerk et al. PRB 67, 165324, '03

Assumption: $G \gg 1$, $I_{Noise} = I_0 + I_n$, $\langle I_0 I_n \rangle = 0$

$\Rightarrow \quad \Delta I^2_{Noise} = \Delta I^2_0 + \Delta I^2_n$ is minimial when:

$$\Delta I^2_0 = \Delta I^2_n = \frac{1}{2} G^2 \Delta I^2_{zp} \overset{\text{impedance-matching}}{=} G^2 \frac{\hbar\omega}{4} g_s \Delta \nu$$

How can this minimum be reached in a given device?

How can this minimum be reached in a given device? $\Delta I_0^2 = \Delta I_n^2 = G^2 \frac{\hbar\omega}{4} g_s \Delta\nu$

$\Delta I_0^2 = \Delta I_0^2(V, T, ...)$, $\Delta I_n^2 = \Delta I_n^2(V, T, \gamma, ...)$, $G = G(V, T, \gamma, ...)$

Enough parameters to achieve noise-balancing (and to stay linear):

$$\Delta I_0^2 = \Delta I_n^2$$

Note: this means $\Delta I_{noise}^2 = 2\Delta I_0^2$

Problem: how to acheive the second equality

without violating the first one?

Needed: a curve in parameter space on which $\frac{\Delta I_0^2}{\Delta I_n^2}$ remains constant.

Answer: keep $\gamma G(V, T, \gamma, ...) = const.$

That's it!

IF qunatum limited performance is achievable, then it will be found on this curve.

Typically, $G \sim \gamma V \Rightarrow$ $\gamma^2 V = const.$

Note : Only measurable quantities are involved!

Why $\gamma^2 V = const.$ maintains noise-balancing $\frac{\Delta I_0^2}{\Delta I_n^2} = 1$?

Typical example, idling noise = shot noise.

electrons passing through the transistor transfer energy to the contacts. The emission spectrum

$\Delta I_0^2 \sim S_0(\omega) \sim$ number of available transitions $\sim V$

Signal source connected - new inelastic transitions involving the degrees of freedom of the signal source.

$I_{in} \to I_{in} + \gamma \delta I_{in}, \quad (\delta I_{in})^2 \sim$ number of available transitions $\sim V$

$GI_{in} \to GI_{in} + \gamma G \delta I_{in}, \quad I_{out} = I_0 + GI_{in} + \gamma G \delta I_{in}.$

Recall $G \sim \gamma V, \quad I_{out} = I_0 + GI_{in} + I_n$

$$\Rightarrow \quad const. = \frac{\Delta I_n^2}{\Delta I_0^2} \sim \frac{\gamma^2 G^2 (\delta I_{in})^2}{V} \sim \gamma^4 V^2$$

$$\Delta I_0^2 = \Delta I_n^2 = G^2 \frac{\hbar\omega_0}{4} g_s \Delta\nu$$

$$\gamma G(V,\gamma) = constant$$

Similar condition, $G \to \infty$, $\gamma \to 0$, $\gamma G = const.$

in parametric amplification theory (Yurke & Denker PRA '84)

with G = pump amplitude

$$\Delta I_0^2 = \Delta I_n^2 = G^2 \frac{\hbar\omega_0}{4} g_s \Delta\nu$$

Noise minimization manual

1. Decouple the input signal from the gate (e.g. by increasing the gate capacitance) and measure the (idling) noise as a function of V.

2. Couple a known (reference) signal to the gate. More noise will appear.

3. Adjust the noise to be *twice* the value of the idling noise. No more. No less .

4. Adjust the gain to match the total noise (in units of ZPF $\hbar\omega_0 g_s \Delta\nu / 2$) while keeping the product γ^2V fixed.

$$\Delta I_0^2 = \Delta I_n^2 = G^2 \frac{\hbar\omega_0}{4} g_l \Delta \nu$$

Limitations

1. Good control of the coupling, a wide linear regime, $G > 1$.

2. Impedance - matching with the load for a wide range of voltages.

3. Applicable for phase - insensitive transistor

4. The transistor should be able to operate close to the Heisenberg limit.

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Limitations

1. Good control of the coupling, a wide linear regime, $G > 1$.

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4. The transistor should be able to operate close to the Heisenberg limit.

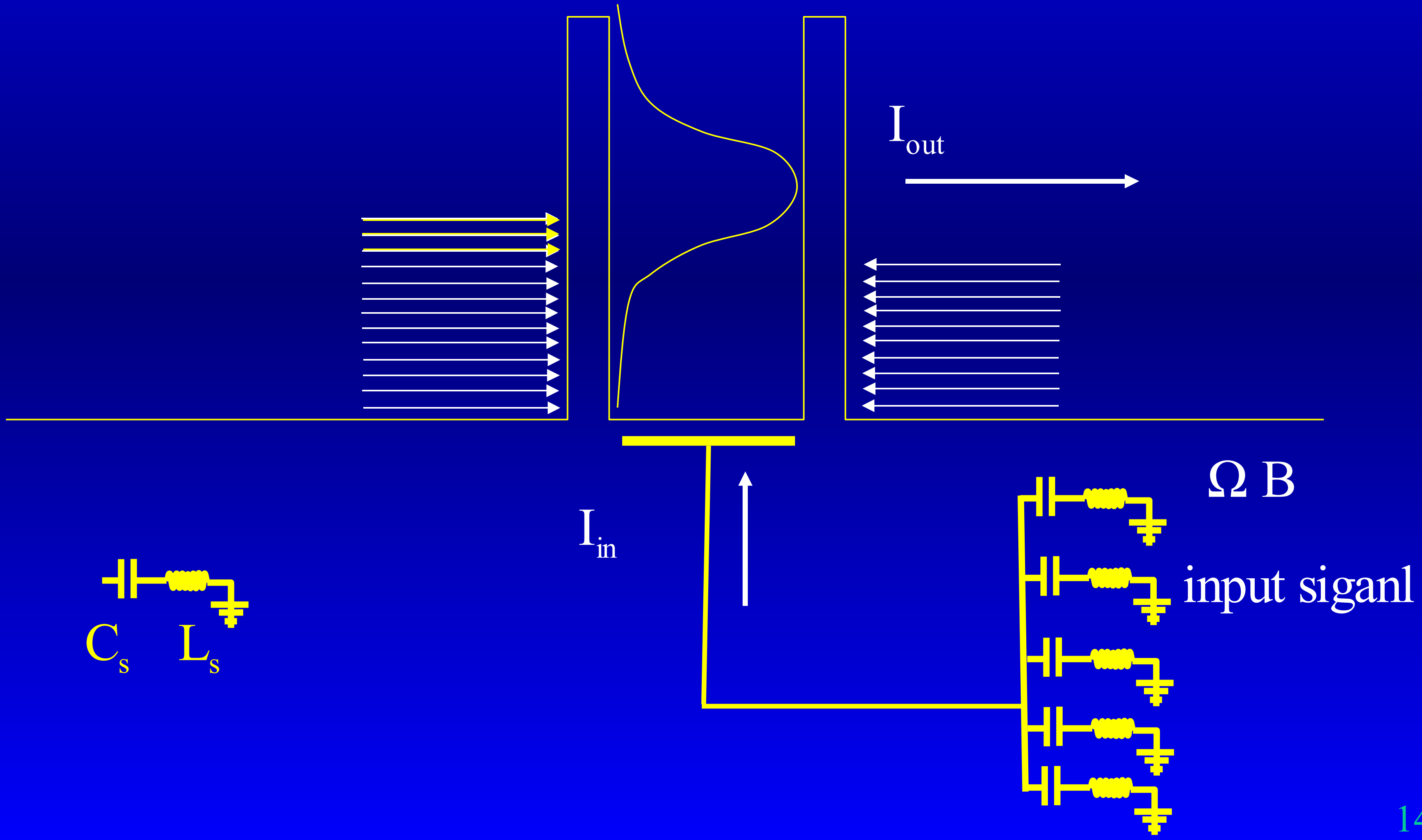
Even if not, the procedure serves as a quality-criterion for quantum noise performance ! Engineering problem or incorrect choice of parameters.

We have a 'black-box' procedure,

let's look inside the box...

Example

Resonant barrier transistor coupled capacitively to an input signal



$$H = \sum_{i=1,2} \int_0^\infty d\varepsilon\, \varepsilon\, b_i^\dagger(\varepsilon) b_i(\varepsilon) + \hbar\omega_A A^\dagger A + \sum_{i=1,2} \int_0^\infty d\varepsilon \frac{ik(\varepsilon)}{\sqrt{2\pi}} \left(b_i^\dagger(\varepsilon) A - A^\dagger b_i(\varepsilon) \right) +$$

$$\int_{B=\omega_0 \pm \Delta\omega/2} d\omega_s \hbar\omega_s a^\dagger(\omega_s) a(\omega_s) + \frac{A^\dagger A e Q_s}{C_g}$$

$$Q_s = \Delta Q_{zp}(\omega_0) \int_B \frac{d\omega_s}{\sqrt{\omega_s}} \left(a^\dagger(\omega_s) + a(\omega_s) \right)$$

$$\Delta Q_{zp}(\omega_0) = \sqrt{\hbar\omega_0 C/2}, \quad \Delta Q_{zp}^2 = 4\frac{\hbar\omega_0}{2} g_s \Delta\nu, \quad \Delta\nu = \frac{\Delta\omega}{2\pi}$$

$$\gamma = \frac{e\Delta Q_{zp}}{C_g k^2} << 1, \qquad H_{int} = \frac{A^\dagger A e Q_s}{C_g} \rightarrow \gamma k^2 \frac{A^\dagger A e Q_s}{\Delta Q_{zp}}$$

$$I_a(t) = \frac{1}{2}\left(\dot{Q}_1(t) - \dot{Q}_2(t) \right), \qquad Q_i(t) = e\int_0^\infty d\varepsilon\, b_i^\dagger(\varepsilon) b_i(\varepsilon)$$

Define. Output: $I_{out}(t) = \frac{1}{2} I_a(t)$, Input: $\tilde{I}_{in}(t) = \frac{1}{2}\omega_s Q_s(t)$

Solving Heisenberg equations of motion in second order in γ we find:

$$I_{out}(t) = I_0(t) + G\sqrt{\frac{g_\ell}{\tilde{g}_s}}\tilde{I}_{in}(t) + I_n(t)$$

$$I_0(t) = \frac{e\hbar}{4\pi}\int_{\pm B} d\omega\, e^{-i\omega t}\int_{-\infty}^{\infty} d\omega'\left(t^*(\omega')b_+^\dagger(\omega')b_-(\omega'+\omega) + t(\omega')b_-^\dagger(\omega'-\omega)b_+(\omega')\right)$$

$$b_\pm(\omega) = \frac{1}{\sqrt{2}}(b_1(\omega) \pm b_2(\omega))$$

$$G = \gamma\frac{eV}{\hbar\omega_0}T\sqrt{2(1-T)}, \quad T=|t|^2, \quad t(\omega) = -k^2/\left(-i\hbar(\omega-\omega_A)+k^2\right)$$

$$g_\ell = \frac{Te^2}{2\pi\hbar},$$

$\tilde{g}_s = \frac{\pi\Delta Q_{zp}^2}{\hbar}$ is the differential linera response of the "current" $\tilde{I}_s(t) = \omega_s Q_s(t)$

$I_n(t)$=.....

Actually: $\hat{G}$ is an operator, but $\Delta\hat{G}^2 << \langle\hat{G}\rangle^2$ in the states we consider

Solving Heisenberg equations of motion in second order in γ we find:

$$I_n(t) = -\gamma^2 \frac{e\hbar^2}{4\pi} \int_{\pm B} d\omega\, e^{-i\omega t} \left[\int_{-\infty}^{\infty} d\omega' \int_{-\infty}^{\infty} d\omega'' \int_B \frac{d\omega_s}{\omega_s} \times \right.$$

$$\sum_{q=\pm 1} \left[q t^*(\omega')t(\omega'')t^*(\omega''-q\omega_s)t^*(\omega'+q\omega_s) b_+^\dagger(\omega'+q\omega_s) b_+^\dagger(\omega''-q\omega_s) b_+(\omega'') b_-(\omega'+\omega) \right.$$

$$\left. + q t(\omega')t^*(\omega'')t(\omega''-q\omega_s)t(\omega'+q\omega_s) b_-^\dagger(\omega'-\omega) b_+^\dagger(\omega'') b_+(\omega''-q\omega_s) b_+(\omega'+q\omega_s) \right]$$

$$-i\gamma^2 \frac{\sqrt{2}e\hbar^2}{2\pi^{3/2}} \int_{-\infty}^{\infty} d\omega' \int_{-\infty}^{\infty} d\omega'' \int_B d\omega_s \times$$

$$\left[t^*(\omega')t(\omega'') \mathrm{P}\int_{-\infty}^{\infty} d\bar{\omega}'\, t^*(\omega''+\bar{\omega}')t^*(\omega'-\bar{\omega}') \times \frac{b_+^\dagger(\omega'-\bar{\omega}') b_+^\dagger(\omega''+\bar{\omega}') b_+(\omega'') b_-(\omega'+\omega)}{\bar{\omega}'^2-\omega_s^2} \right.$$

$$\left.\left. - t(\omega')t^*(\omega'') \mathrm{P}\int_{-\infty}^{\infty} d\bar{\omega}'\, t(\omega''+\bar{\omega}')t(\omega'-\bar{\omega}') \times \frac{b_-^\dagger(\omega'-\omega) b_+^\dagger(\omega'') b_+(\omega''+\bar{\omega}') b_+(\omega'-\bar{\omega}')}{\bar{\omega}'^2-\omega_s^2} \right] \right]$$

Noise spectra:

$$\Delta I_0^2(t) = T(1-T)\frac{e^3V}{4\pi\hbar}\Delta\nu\,, \quad \Delta I_n^2(t) = \gamma^4 T^5(1-T)\left(\frac{eV}{\hbar\omega_0}\right)^2\frac{e^3V}{\pi\hbar}\Delta\nu$$

The spectra are uncorrelated (at least for large eV): $\langle I_0 I_n\rangle = 0$

From the above: $\Delta I_0(t)\Delta I_n(t) = \frac{1}{4}G^2\hbar\omega_0 g_\ell\Delta\nu$

Require: $\Delta I_0(t) = \Delta I_n(t)$ Then: $\gamma^2\frac{eV}{\hbar\omega_0}T^2 = 1$

Moving along this constraint we keep γG=const as expected and we can obtain the desired gain. Recalling the expression for G one finds: $G^H = \sqrt{2(1-T)}/(\gamma T)$

assumptions about T are exact only for T=3/4, thus

$$G^H = \frac{2\sqrt{2}}{3}\frac{1}{\gamma}$$

Note : This is only a check. The procedure is a black-box procedure!

To summarize, we presented an experimental procedure for approaching the optimum noise performance allowed by quantum mechanics for linear transistor amplifiers.

Our procedure does not require theoreitcal modelling of the system.

It does require the transistor to be able to operate close to the Heisenberg limit. In case it does not, the procedure serves as a criterion for the quality of the transistor quantum noise perormance.