Nanophysics: from fundamentals to applications
Hanoi, August 6-12, 2006

Multiparticle Aharonov-Bohm Effects: A quantum interference of *N* identical particles

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Refs.: H.-S. Sim and E. V. Sukhorukov, PRL 96, 020407 (2006).

Outline

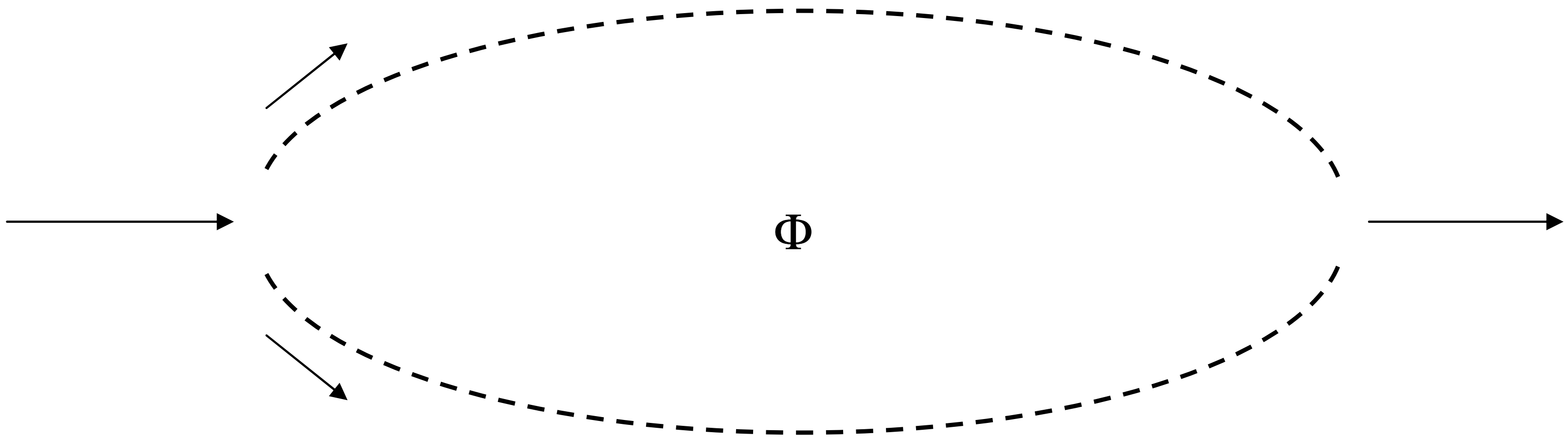
- **Single-particle quantum interference:**
  - Aharonov-Bohm effects

- **Two-particle interference:**
  - Entanglement
  - Hanbury Brown-Twiss two-photon interference

- **Prediction of *N*-particle Aharonov-Bohm effect:**
  - one of the simplest *N*-particle interferences
  - full counting statistics
  - Greenberger-Horne-Zeilinger (GHZ) entanglement
  - multiparticle quantum nonlocality

Introduction: Single-particle interference

Single-particle Aharonov-Bohm (AB) effect

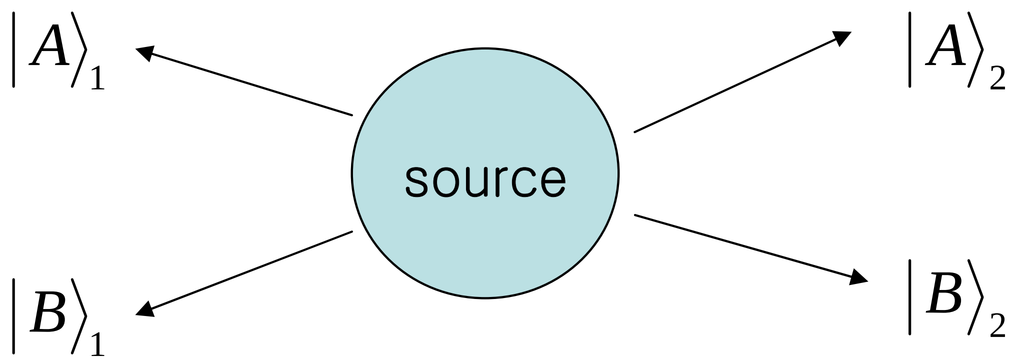


$$I_R = \Gamma_{BS} \cos\left(2\pi \frac{\Phi}{\phi_0} + \sum \phi_i \right)$$

Note: Young's slit experiments

Introduction: Entanglement

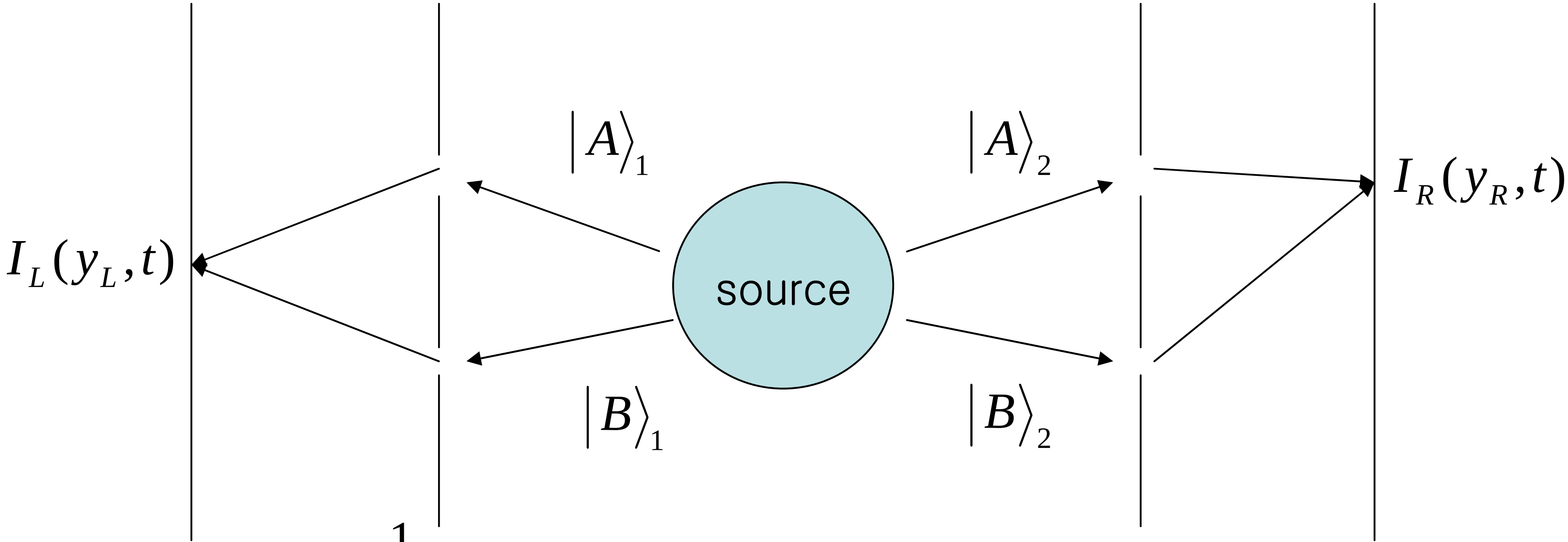
Entanglement: a composite system



$$\psi = \frac{1}{\sqrt{2}}\left(|A\rangle_1 |B\rangle_2 + |B\rangle_1 |A\rangle_2\right)$$

- **Wave function $\neq$ single direct product of single-particle states.**
  (ex.) spin singlet
- **Multiparticle observables such as shot noise are need for detection.**
- **Violation of Bell inequalities. Quantum nonlocality.**
- **Quantum information and quantum computation.**

Introduction: Two-particle interference in optics



$$\psi = \frac{1}{\sqrt{2}}(|A\rangle_1 |B\rangle_2 + |B\rangle_1 |A\rangle_2)$$

$$\langle y_L, y_R | \psi \rangle = \frac{1}{\sqrt{2}}(e^{ikL_{A1}(y_L)} e^{ikL_{B2}(y_R)} + e^{ikL_{A2}(y_R)} e^{ikL_{B1}(y_L)})$$

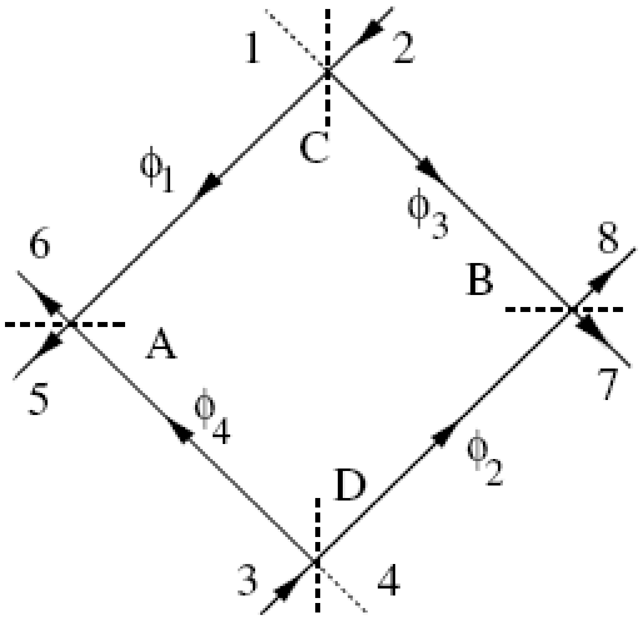
- **Two-particle interference (Hanbury-Brown Twiss correlation)**

$$\langle I_L(y_L,t) I_R(y_R,t) \rangle \sim |\langle y_L, y_R | \psi \rangle|^2 \sim \cos(k\theta(y_L - y_R) + \delta)$$

- **No interference in single-particle observables** $\langle I_{L/R}(y_{L/R},t) \rangle$

Two-particle interference in mesoscopics

P. Samuelsson, E. V. Sukhorukov, and M. Büttiker, Phys. Rev. Lett. **92**, 026805 (2004).

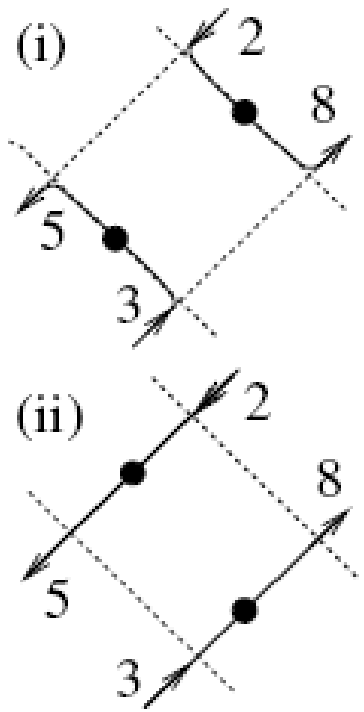


$$P_{\alpha\beta}(\omega) = 2\int_{-\infty}^{\infty} dt e^{i\omega t}\left\langle \delta I_{\alpha}(t+t_0)\delta I_{\beta}(t_0)\right\rangle$$

$$S_{58} = -(e^2/4h)|eV|[1+\cos(\phi_1+\phi_2-\phi_3-\phi_4)]$$

- **No interference in average currents**
- **Quantum nonlocality**

Orbital Entanglement
(postselection)



Basic question:
N-particle Interference?

- ***A good example of N-particle interference?***

- ***N-particle entanglement?***

- ***Does multiparticle interference mean multiparticle nonlocality?***

Detailed outline

- To answer the questions, we propose *N*-particle Hanbury Brown-Twiss interferometer, which encloses magnetic flux, using a mesoscopic system. The interferometer shows the following interesting features.

- Multiparticle Aharonov-Bohm (AB) interference:

$$\left\langle \delta I_{D1} \delta I_{D2} \ldots \delta I_{DN} \right\rangle \propto \cos\left(2\pi \frac{\Phi}{\phi_0} + \sum \phi_i \right)$$

$$\left\langle \delta I_{D1} \delta I_{D2} \ldots \delta I_{Dj} \right\rangle \longrightarrow$$ No oscillations for $j < N$

- Origin of interference: Greenberger-Horne-Zeilinger (GHZ) entanglement

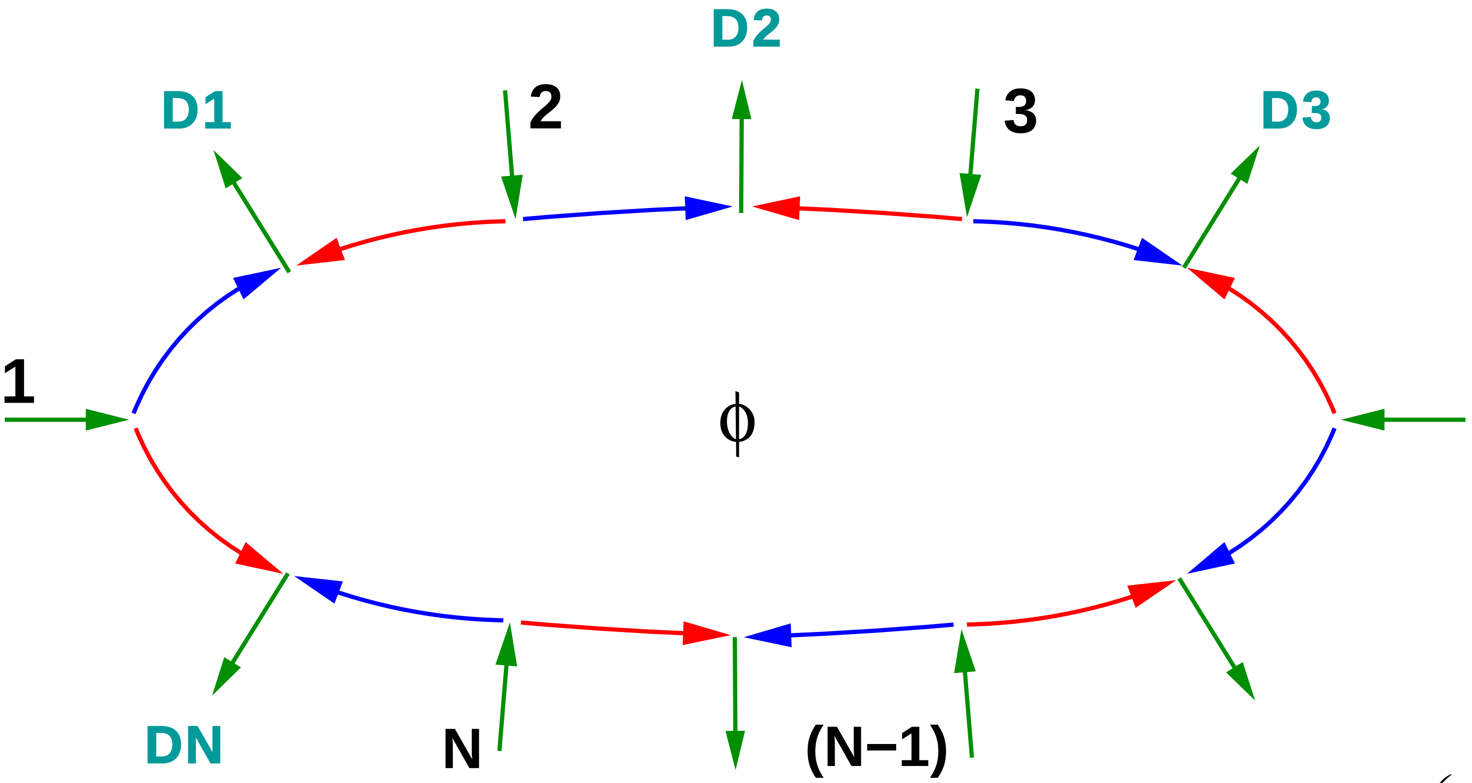
$$|\psi_N\rangle = p e^{i\varphi_p} |\uparrow \cdots \uparrow\rangle + q e^{i\varphi_q} |\downarrow \cdots \downarrow\rangle$$

\- Simplest multiparticle interference!

- Multiparticle coherence and nonlocality:

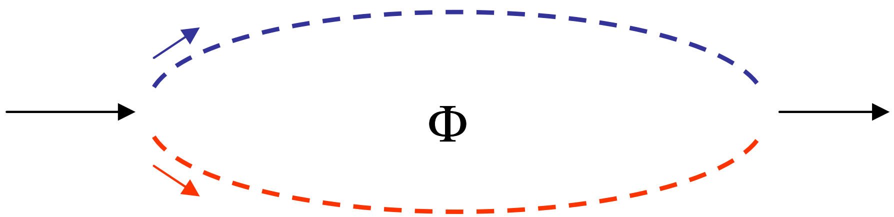
\- Sufficiently strong AB visibility $\longrightarrow$ *N*-particle quantum nonlocality!

Prediction: Multiparticle AB interference



$$\left\langle \delta I_{D1}\delta I_{D2}\ldots\delta I_{DN}\right\rangle \propto \cos\left(2\pi\frac{\Phi}{\phi_0}+\sum\phi_i\right)$$

$$|\psi_N\rangle = pe^{i\varphi_p}|\uparrow\cdots\uparrow\rangle + qe^{i\varphi_q}|\downarrow\cdots\downarrow\rangle$$



$$I_R \propto \cos\left(2\pi\frac{\Phi}{\phi_0}+\sum\phi_i\right)$$

N-particle HBT interferometer

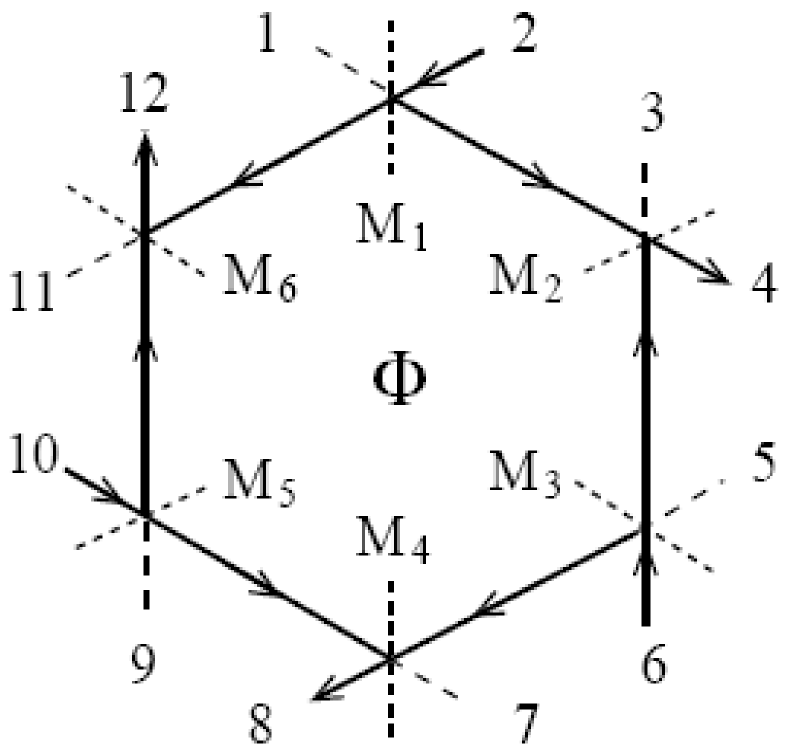


FIG. 1: Left: A three-particle Hanbury Brown-Twiss (HBT) setup with $N = 3$ independent electron sources (reservoirs 1, 5, 9), $2N$ beam splitters (M_i), $2N$ chiral current paths, each connecting two beam splitters, and $2N$ detectors (reservoirs 3, 4, 7, 8, 11, 12). Arrows indicate electron trajectories. In the setup, any single electron can not enclose magnetic flux Φ, while the $2N$ current paths together do. For example, electrons incident from reservoir 1 move to detectors 3 or 4 (11 or 12) via the path between splitters M_1 and M_2 (M_6).

Three-particle HBT setup

Sources: 2, 6, 10
Detectors: 4, 8, 12, for examples

Transmission probability at M_i T_i

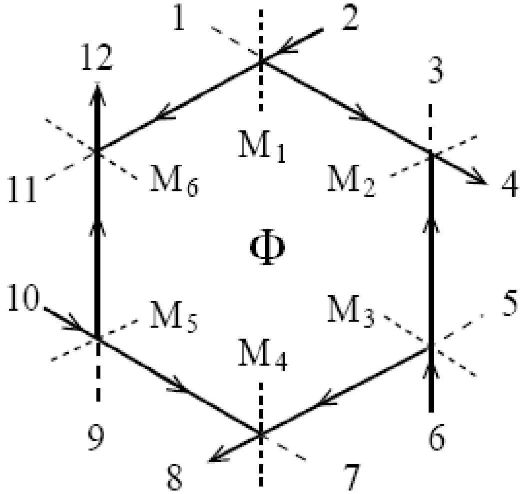
$$R_i = 1 - T_i$$

Phase accumulation at path i ϕ_i

N-particle AB effect

N-th order cumulant of number cross correlation: (zero temperature limit)

$$Q_{\alpha_1,\alpha_2,\cdots,\alpha_N} = 2\,\mathrm{sign}(\alpha_1,\alpha_2,\cdots,\alpha_N)\Gamma_{\mathrm{BS}}\cos(\phi_{\mathrm{tot}})$$



(ex) detector indices: $\alpha_1 = 4, \alpha_2 = 8, \alpha_3 = 12$

$$\Gamma_{\mathrm{BS}} = \prod_{i=1}^{2N} \sqrt{T_i R_i}$$

$$\phi_{\mathrm{tot}} = 2\pi\Phi/\Phi_0 + \sum_{i=1}^{2N} \phi_i$$

$$\mathrm{sign}(\alpha_1,\alpha_2,\cdots,\alpha_N) = (-1)^{N-1+\sum_i \alpha_i}$$

Exchange effect (Fermionic statistics)

Detector phase

All the *(2<J<N)*-th order cumulants vanish!

The 1st and the 2nd order cumulants: No oscillation!

$$Q_{\alpha_i} = P_{\alpha_i} R_{2i-1} + (1-P_{\alpha_i}) T_{2i+1}$$

$$Q_{\alpha_i,\alpha_{i+1}} = -P_{\alpha_{i+1}}(1-P_{\alpha_i}) T_{2i+1} R_{2i+1}$$

$P_{\alpha_i} = R_{2i}$ ($P_{\alpha_i} = T_{2i}$) for α_i being odd (even)

Multiparticle GHZ entanglement

Initial multiparticle states: $|\Psi\rangle = \prod_{0<E<eV} \prod_{i=1}^{N} c_{4i-2}^{\dagger}(E)|0\rangle$

After passing beam splitters :

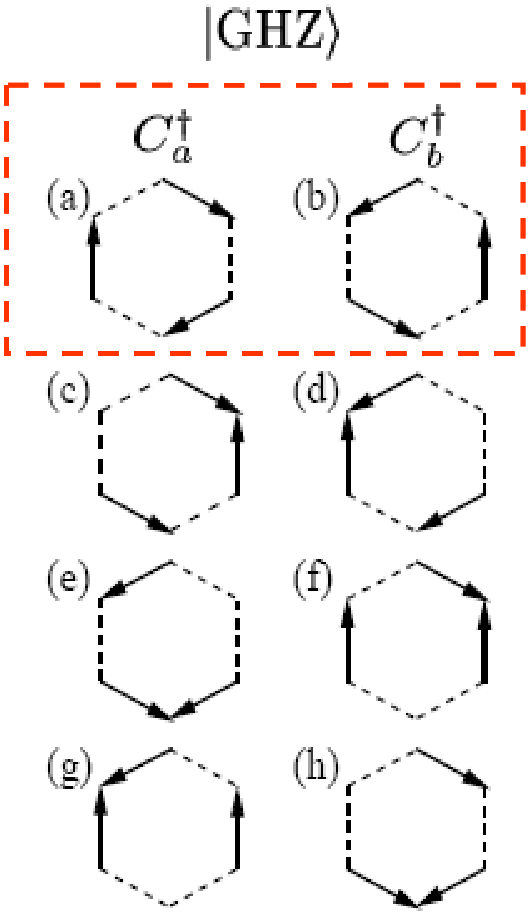
$$c_{4i-2}^{\dagger} = -i\sqrt{R_{2i-1}}\, a_{i+1}^{\dagger} + \sqrt{T_{2i-1}}\, b_i^{\dagger}$$

$$|\Psi\rangle = \prod_{0<E<eV} [\gamma_a C_a^{\dagger}(E) + \gamma_b C_b^{\dagger}(E) + D^{\dagger}(E)]|0\rangle$$

$$\gamma_a = \prod_{i=1}^{N}(-i\sqrt{R_{2i-1}}) \qquad \gamma_b = -\prod_{i=1}^{N}\sqrt{T_{2i-1}}$$

$C_a^{\dagger} = \prod_{i=1}^{N} a_i^{\dagger}$ Clockwise rotating N electrons

$C_b^{\dagger} = \prod_{i=1}^{N} b_i^{\dagger}$ Anticlockwise rotating N electrons



GHZ entanglement :

$$|\psi_N\rangle = (\gamma_a C_a^{\dagger} + \gamma_b C_b^{\dagger})|0\rangle = pe^{i\theta_p}|\uparrow\cdots\uparrow\rangle + qe^{i\theta_q}|\downarrow\cdots\downarrow\rangle$$

$$p^2 = 1 - q^2 = |\gamma_a|^2/(|\gamma_a|^2 + |\gamma_b|^2) \qquad \theta_p = -\pi N/2 \quad \theta_q = \pi$$

Postselection of GHZ

Joint detection probability (JDP) of *N* particles:

$$\mathcal{P}_{\{\alpha\}} \propto \langle \Psi | I_{\alpha_1} I_{\alpha_2} \cdots I_{\alpha_N} | \Psi \rangle$$

$$\sum_{\{\alpha\}} \mathcal{P}_{\{\alpha\}} = 1$$

$$I_{\alpha_i} = (e/2\pi\hbar) \iint dE dE' c^\dagger_{\alpha_i}(E) c_{\alpha_i}(E')$$

Current operator at the same time t=0

$$\mathcal{P}_{\{\alpha\}} \propto \boxed{\prod_{i=1}^{N} R_{2i-1} P_{\alpha_i} + \prod_{i=1}^{N} T_{2i-1}(1 - P_{\alpha_i})} + Q_{\{\alpha\}}$$

$$\langle \uparrow\uparrow \ldots \uparrow | I_{\alpha 1} I_{\alpha 2} \ldots | \uparrow\uparrow \ldots \uparrow \rangle + \langle \downarrow\downarrow \ldots \downarrow | I_{\alpha 1} I_{\alpha 2} \ldots | \downarrow\downarrow \ldots \downarrow \rangle$$

$$\langle \uparrow\uparrow \ldots \uparrow | I_{\alpha 1} I_{\alpha 2} \ldots | \downarrow\downarrow \ldots \downarrow \rangle + \langle \downarrow\downarrow \ldots \downarrow | I_{\alpha 1} I_{\alpha 2} \ldots | \uparrow\uparrow \ldots \uparrow \rangle$$

The joint detection postselects the GHZ entanglement part from the Fermi sea.

Bell measurement: pseudospin GHZ = spin GHZ

***N*-spin GHZ states**: $|\psi_N\rangle = pe^{i\varphi_p}|\uparrow\cdots\uparrow\rangle + qe^{i\varphi_q}|\downarrow\cdots\downarrow\rangle$

Bell parameter:
Spin cross-correlation

$$E_N = \langle \sigma_{\vec{n}_1} \otimes \cdots \otimes \sigma_{\vec{n}_N} \rangle$$

$$\boxed{E_N = g(p,q)\prod_{i=1}^{N}\cos\delta_i + 2pq\cos\theta\prod_{i=1}^{N}\sin\delta_i}$$

$g(p,q) = p^2 + (-1)^N q^2$ and $\varphi = \varphi_p - \varphi_q + \sum_{j=1}^{N}\varphi_j$

***N*-pseudospin**
GHZ in HBT:

$$|\psi_N\rangle = (\gamma_a C_a^\dagger + \gamma_b C_b^\dagger)|0\rangle$$

***Pseudospin rotation*:**

$$\cos\delta_i = T_{2i} - R_{2i} \qquad \theta_i = \phi_{2i-1} + \phi_{2i} - \pi/2$$

Bell parameter:
Pseudospin cross correlation

$$E_N = \sum_{\{\alpha\}}(-1)^{\sum_{i=1}^{N}\alpha_i} P_{\{\alpha\}}$$

The Bell parameter of our pseudospin GHZ state

= that of spin GHZ.

Multiparticle nonlocality

***N*-particle Bell operator**：

$$M_N = \frac{1}{2}(\sigma_{\vec{n}_N} + \sigma_{\vec{n}'_N}) \otimes M_{N-1} + \frac{1}{2}(\sigma_{\vec{n}_N} - \sigma_{\vec{n}'_N}) \otimes M'_{N-1}$$

Mermin-Ardehali-Belinskii-Klyshko inequality: $\langle M_N \rangle \leq 1$

Svetlichny inequality: $\langle S_N \rangle \leq 2^{(N-2)/2}$

$$S_N = (1/\sqrt{2})(M_N + M'_N) \text{ for } N \text{ being odd}$$
$$S_N = M_N \text{ otherwise}$$

HBT pseudospin GHZ state violates the Bell's inequalities when

$$\underline{V_{\mathrm{AB}}} > 1/\sqrt{2}$$

Visibility of JDP

Thus, the sufficiently strong visibility of the interference directly means multiparticle nonlocality.

Concluding Remark:

- GOOD example of N-particle interference.

- N-particle interference
  - origin: a specific N-particle entanglement such as GHZ
  - detection: full counting statistics
  - tool for studying N-particle nonlocality

Refs.: H.-S. Sim and E. V. Sukhorukov, PRL 96, 020407 (2006).