Two-photon Rabi Oscillations of Biexciton in Semiconductor Quantum Dot

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Abstract

Theory of Rabi oscillations in a three-level system "ground state-exciton-biexciton" in a two-level semiconductor quantum dot is presented. In particular the two-photon Rabi oscillations of biexciton is studied in details. The Rabi flopping of the populations of different states as well as the Rabi splitting of the spontaneous emission spectral lines are investigated.

Outline

I. Introduction

II. State Vectors and Energy Spectrum of the Electron System

III. Green Functions and Rabi Oscillations

IV. Rabi Flopping of Populations

V. Rabi Splitting in Photon Emission Spectra

VI. Conclusion and Discussions

I. INTRODUCTION

The Rabi oscillations in the semiconductor QDs

were studied in many works.

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II. State Vectors and Energy Spectrum of the Electron System

The Model:

The simplest model of the semiconductor QD's disk-shaped direct band gap semiconductor QD with two discrete energy levels,

upper level conduction

lower one heavy-hole valence band

$$\left(J = \frac{3}{2} \ \text{ and } \ J_z = \pm\frac{3}{2} \right)$$

Hamiltonian of the electron system in the QD:

$$H_{dot} = \sum_{i=1,2}[E_i^0 c_i^+ c_i + U_i n_{i\uparrow} n_{i\downarrow}] + U_{12} n_1 n_2 + U_{ex}^{(z)} s_{1z} s_{2z} + U_{ex}^{(x,y)}[s_{1x} s_{2x} + s_{1y} s_{2y}],$$

$$c_i = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix}, \qquad E_1^0 > E_2^0,$$

$$n_{i\sigma} = c_{i\sigma}^+ c_{i\sigma}, \qquad n_i = n_{i\uparrow} + n_{i\downarrow},$$

$$s_{i\alpha} = \frac{1}{2} c_i^+ \sigma_\alpha c_i,$$

16 Different States

- The vacuum state $|0\rangle$
- Four one-electron states $\Phi_i^{\sigma} = c_{i\sigma}^{+}|0\rangle, \ \sigma = \uparrow, \downarrow, \ i = 1,2$
- Six two-electron states

$$\Phi_{11} = c_{1\uparrow}^{+} c_{1\downarrow}^{+}|0\rangle \qquad \Phi_{22} = c_{2\uparrow}^{+} c_{2\downarrow}^{+}|0\rangle$$

$$\Phi_{12}^{\sigma} = c_{1\sigma}^{+} c_{2\sigma}^{+}|0\rangle, \quad \sigma = \uparrow, \downarrow,$$

$$\Phi_{12}^{t} = \frac{1}{\sqrt{2}}\left(c_{1\uparrow}^{+} c_{2\downarrow}^{+} + c_{1\downarrow}^{+} c_{2\uparrow}^{+}\right)|0\rangle,$$

$$\Phi_{12}^{s} = \frac{1}{\sqrt{2}}\left(c_{1\uparrow}^{+} c_{2\downarrow}^{+} - c_{1\downarrow}^{+} c_{2\uparrow}^{+}\right)|0\rangle,$$

- Four three-electron states

$$\Phi_{112}^{\sigma} = c_{1\uparrow}^{+} c_{1\downarrow}^{+} c_{2\sigma}^{+} \left| 0 \right\rangle, \qquad \sigma = \uparrow, \downarrow,$$

$$\Phi_{122}^{\sigma} = c_{1\sigma}^{+} c_{2\uparrow}^{+} c_{2\downarrow}^{+} \left| 0 \right\rangle, \qquad \sigma = \uparrow, \downarrow,$$

- One four-electron state

$$\Phi_{1122} = c_{1\uparrow}^{+} c_{1\downarrow}^{+} c_{2\uparrow}^{+} c_{2\downarrow}^{+} \left| 0 \right\rangle.$$

The electromagnetic radiation is linearly polarized in the direction of the axis Ox

$$H_{em} = \lambda\left(e^{i\omega_0 t} c_2^+ c_1 + e^{-i\omega_0 t} c_1^+ c_2\right).$$

$$H = H_{dot} + H_{em}.$$

Unitary transformation

$$U = e^{i\frac{\omega_0 t}{2}\left(c_2^+ c_2 - c_1^+ c_1\right)},$$

$$\widetilde{H} = U^+ H U + i\frac{dU^+}{dt}U.$$

$$\tilde{H}_{dot} = \sum_{i=1,2}[E_i c_i^+ c_i + U_i n_{i\uparrow} n_{i\downarrow}] + U_{12} n_1 n_2 + U_{ex}^{(z)} s_{1z} s_{2z} +$$

$$+ U_{ex}^{(x,y)} [s_{1x} s_{2x} + s_{1y} s_{2y}],$$

$$\tilde{H} = \sum_{i=1,2}[E_i c_i^+ c_i + U_i n_{i\uparrow} n_{i\downarrow}] + U_{12} n_1 n_2 + U_{ex}^{(z)} s_{1z} s_{2z} +$$

$$+ U_{ex}^{(x,y)} [s_{1x} s_{2x} + s_{1y} s_{2y}] + \lambda (c_2^+ c_1 + c_1^+ c_2).$$

$$E_1 = E_1^0 - \frac{\omega_0}{2}, \quad E_2 = E_2^0 + \frac{\omega_0}{2} \cdot$$

The Hamiltonian generates the mixing between following eigenstates of the Hamiltonian $\tilde{H}_{dot}$:

$$\Phi_1^{\sigma} \leftrightarrow \Phi_2^{\sigma} ,$$

$$\Phi_{11} \leftrightarrow \Phi_{12}^{s} \leftrightarrow \Phi_{22} ,$$

$$\Phi_{112}^{\sigma} \leftrightarrow \Phi_{122}^{\sigma} ,$$

$$\sigma = \uparrow , \downarrow .$$

Two pairs of one-electron states and two pairs of three-electron states are four two-level systems.

The set of three two-electron states

$$\Phi_{11}\,,\ \Phi_{12}^{s}\quad \text{and}\quad \Phi_{22}$$

with energies E_{11} , E_{12}^{s} and E_{22}

is a three-level system

(ground state, singlet-exciton, biexciton)

Φ_{12}^{t} is a separate two-electron state.

The influence of the biexciton on the optical resonance of the exciton was discussed in many works

1. T. H. Stievater, *et. al.* , Phys. Rev. Lett. **87** (2001), 133603.
2. P. Borri, *et. al.* , Phys. Rev. B **66** (2002), 081306.
3. S. Stufler, *et. al.* , , Phys. Rev. B **72** (2005), 121301(R).
4. R. W. Helmes, *et. al.* , Phys. Rev. B **72** (2005), 125301.
5. J. M. Villas-Bôas, *et. al.* , Phys. Rev. Lett. **94** (2005), 057404; cond-mat/0509731.

The two-photon optical resonance of the biexciton was observed in a recent experiment

1. S. Stufler, *at. al.* , Phys. Rev. B **73** (2006), 125304.

III. Green Functions and Rabi Oscillations

Two-electron states: Two-particle Green functions

$$G^{ijkl}(t-t') = -i\theta(t-t')\left\langle 0 \middle| c_{i\sigma}(t)c_{j-\sigma}(t)c^{+}_{k-\sigma}(t')c^{+}_{l\sigma}(t') \middle| 0 \right\rangle ,$$

$$i,\ j,\ k,\ l = 1,\ 2,$$

$F^{ijkl}(\omega)$ the Fourier transforms,

$$G^{ijkl}(t-t') = \frac{1}{2\pi}\int e^{-i\omega(t-t')}F^{ijkl}(\omega)d\omega .$$

$$F^{1111}(\omega)=\frac{(\tilde{E}_{11}-E^{s}_{12})(\tilde{E}_{11}-E_{22})-2\lambda^2}{(\tilde{E}_{11}-\tilde{E}_{12})(\tilde{E}_{11}-\tilde{E}_{22})}\frac{1}{\omega-\tilde{E}_{11}}$$

$$+\frac{(\tilde{E}_{12}-E^{s}_{12})(\tilde{E}_{12}-E_{22})-2\lambda^2}{(\tilde{E}_{12}-\tilde{E}_{11})(\tilde{E}_{12}-\tilde{E}_{22})}\frac{1}{\omega-\tilde{E}_{12}}$$

$$+\frac{(\tilde{E}_{22}-E^{s}_{12})(\tilde{E}_{22}-E_{22})-2\lambda^2}{(\tilde{E}_{22}-\tilde{E}_{11})(\tilde{E}_{22}-\tilde{E}_{12})}\frac{1}{\omega-\tilde{E}_{22}}\quad ,$$

$$F^{2222}(\omega)=\frac{(\tilde{E}_{11}-E_{11})(\tilde{E}_{11}-E^{s}_{12})-2\lambda^2}{(\tilde{E}_{11}-\tilde{E}_{12})(\tilde{E}_{11}-\tilde{E}_{22})}\cdot\frac{1}{\omega-\tilde{E}_{11}}$$

$$+\frac{(\tilde{E}_{12}-E_{11})(\tilde{E}_{12}-E^{s}_{12})-2\lambda^2}{(\tilde{E}_{12}-\tilde{E}_{11})(\tilde{E}_{12}-\tilde{E}_{22})}\cdot\frac{1}{\omega-\tilde{E}_{12}}$$

$$+\frac{(\tilde{E}_{22}-E_{11})(\tilde{E}_{22}-E^{s}_{12})-2\lambda^2}{(\tilde{E}_{22}-\tilde{E}_{11})(\tilde{E}_{22}-\tilde{E}_{12})}\cdot\frac{1}{\omega-\tilde{E}_{22}}\quad ,$$

$$F^{1122}(\omega) = F^{2211}(\omega) = \frac{2\lambda^2}{(\tilde{E}_{11} - \tilde{E}_{12})(\tilde{E}_{11} - \tilde{E}_{22})} \frac{1}{\omega - \tilde{E}_{11}}$$

$$+ \frac{2\lambda^2}{(\tilde{E}_{12} - \tilde{E}_{11})(\tilde{E}_{12} - \tilde{E}_{22})} \frac{1}{\omega - \tilde{E}_{12}}$$

$$+ \frac{2\lambda^2}{(\tilde{E}_{22} - \tilde{E}_{11})(\tilde{E}_{22} - \tilde{E}_{12})} \frac{1}{\omega - \tilde{E}_{22}} \ ,$$

$$F^{1212}(\omega) + F^{1221}(\omega) = F^{2112}(\omega) + F^{2121}(\omega) =$$

$$= F^{1212}(\omega) + F^{2112}(\omega) = F^{1221}(\omega) + F^{2121}(\omega) =$$

$$= \frac{(\tilde{E}_{11} - E_{11})(\tilde{E}_{11} - E_{22})}{(\tilde{E}_{11} - \tilde{E}_{12})(\tilde{E}_{11} - \tilde{E}_{22})} \frac{1}{\omega - \tilde{E}_{11}} + \frac{(\tilde{E}_{12} - E_{11})(\tilde{E}_{12} - E_{22})}{(\tilde{E}_{12} - \tilde{E}_{11})(\tilde{E}_{12} - \tilde{E}_{22})} \frac{1}{\omega - \tilde{E}_{12}}$$

$$+ \frac{(\tilde{E}_{22} - E_{11})(\tilde{E}_{22} - E_{22})}{(\tilde{E}_{22} - \tilde{E}_{11})(\tilde{E}_{22} - \tilde{E}_{12})} \frac{1}{\omega - \tilde{E}_{22}} \ ,$$

$$F^{1211}(\omega) = F^{2111}(\omega) = F^{1112}(\omega) = F^{1121}(\omega) =$$

$$= \lambda \frac{\tilde{E}_{11} - E_{22}}{(\tilde{E}_{11} - \tilde{E}_{12})(\tilde{E}_{11} - \tilde{E}_{22})} \frac{1}{\omega - \tilde{E}_{11}} + \lambda \frac{\tilde{E}_{12} - E_{22}}{(\tilde{E}_{12} - \tilde{E}_{11})(\tilde{E}_{12} - \tilde{E}_{22})} \frac{1}{\omega - \tilde{E}_{12}}$$

$$+ \lambda \frac{\tilde{E}_{22} - E_{22}}{(\tilde{E}_{22} - \tilde{E}_{11})(\tilde{E}_{22} - \tilde{E}_{12})} \frac{1}{\omega - \tilde{E}_{22}} \ ,$$

$$F^{1222}(\omega) = F^{2122}(\omega) = F^{2212}(\omega) = F^{2221}(\omega) =$$

$$= \lambda \frac{\tilde{E}_{11} - E_{11}}{(\tilde{E}_{11} - \tilde{E}_{12})(\tilde{E}_{11} - \tilde{E}_{22})} \frac{1}{\omega - \tilde{E}_{11}} + \lambda \frac{\tilde{E}_{12} - E_{11}}{(\tilde{E}_{12} - \tilde{E}_{11})(\tilde{E}_{12} - \tilde{E}_{22})} \frac{1}{\omega - \tilde{E}_{12}}$$

$$+ \lambda \frac{\tilde{E}_{22} - E_{11}}{(\tilde{E}_{22} - \tilde{E}_{11})(\tilde{E}_{22} - \tilde{E}_{12})} \frac{1}{\omega - \tilde{E}_{22}} \ .$$

The poles $\tilde{E}_{11}$, $\tilde{E}_{12}$ and $\tilde{E}_{22}$

are three roots of the equation

$$(\omega - E_{11})(\omega - E^s_{12})(\omega - E_{22}) - 2\lambda^2(2\omega - E_{11} - E_{22}) = 0\,.$$

They are the eigenvalues of the total Hamiltonian.

Chose the notations such that

$$\tilde{E}_{11} \to E_{11}\,, \quad \tilde{E}_{12} \to E^s_{12}\,, \quad \tilde{E}_{22} \to E_{22} \ \text{ at } \lambda \to 0.$$

Denote

$$\Phi_{11}, \Phi_{12}^{s} \text{ and } \Phi_{22}$$

the eigenstate of $\tilde{H}_{dot}$ (in the absence of the radiation)

$$\tilde{\Phi}_{11}, \tilde{\Phi}_{12} \text{ and } \tilde{\Phi}_{22}$$

the corresponding eigenstates of $\tilde{H}$

(in the presence of the radiation)

Between two systems $\Phi_{11}, \Phi_{12}^{s}, \Phi_{22}$ and $\tilde{\Phi}_{11}, \tilde{\Phi}_{12}, \tilde{\Phi}_{22}$

there exists some unitary transformation:

$$\Phi_{11} = a_{11}\tilde{\Phi}_{11} + a_{12}\tilde{\Phi}_{12} + a_{13}\tilde{\Phi}_{22} \, ,$$

$$\Phi^{s}_{12} = a_{21}\tilde{\Phi}_{11} + a_{22}\tilde{\Phi}_{12} + a_{23}\tilde{\Phi}_{22} \, ,$$

$$\Phi_{22} = a_{31}\tilde{\Phi}_{11} + a_{32}\tilde{\Phi}_{12} + a_{33}\tilde{\Phi}_{22} \, ;$$

$$\tilde{\Phi}_{11} = a_{11}\Phi_{11} + a_{21}\Phi^{s}_{12} + a_{31}\Phi_{22} \, ,$$

$$\tilde{\Phi}_{12} = a_{12}\Phi_{11} + a_{22}\Phi^{s}_{12} + a_{32}\Phi_{22} \, ,$$

$$\tilde{\Phi}_{22} = a_{13}\Phi_{11} + a_{23}\Phi^{s}_{12} + a_{33}\Phi_{22} \, .$$

Green functions are expressed in terms of the coefficients $a_{\alpha\beta}$.

Result of the comparison:

$$a_{11}^2 = \frac{(\tilde{E}_{11} - E_{12}^s)(\tilde{E}_{11} - E_{22}) - 2\lambda^2}{(\tilde{E}_{11} - \tilde{E}_{12})(\tilde{E}_{11} - \tilde{E}_{22})} \quad , \qquad a_{21}^2 = \frac{(\tilde{E}_{11} - E_{11})(\tilde{E}_{11} - E_{22})}{(\tilde{E}_{11} - \tilde{E}_{12})(\tilde{E}_{11} - \tilde{E}_{22})} \quad ,$$

$$a_{12}^2 = \frac{(\tilde{E}_{12} - E_{12}^s)(\tilde{E}_{12} - E_{22}) - 2\lambda^2}{(\tilde{E}_{12} - \tilde{E}_{11})(\tilde{E}_{12} - \tilde{E}_{22})} \quad , \qquad a_{22}^2 = \frac{(\tilde{E}_{12} - E_{11})(\tilde{E}_{12} - E_{22})}{(\tilde{E}_{12} - \tilde{E}_{11})(\tilde{E}_{12} - \tilde{E}_{22})} \quad ,$$

$$a_{13}^2 = \frac{(\tilde{E}_{22} - E_{12}^s)(\tilde{E}_{22} - E_{22}) - 2\lambda^2}{(\tilde{E}_{22} - \tilde{E}_{11})(\tilde{E}_{22} - \tilde{E}_{12})} \quad ; \qquad a_{23}^2 = \frac{(\tilde{E}_{22} - E_{11})(\tilde{E}_{22} - E_{22})}{(\tilde{E}_{22} - \tilde{E}_{11})(\tilde{E}_{22} - \tilde{E}_{12})} \quad ;$$

$$a_{31}^2 = \frac{(\tilde{E}_{11} - E_{11})(\tilde{E}_{11} - E_{12}^s) - 2\lambda^2}{(\tilde{E}_{11} - \tilde{E}_{12})(\tilde{E}_{11} - \tilde{E}_{22})} \quad ,$$

$$a_{32}^2 = \frac{(\tilde{E}_{12} - E_{11})(\tilde{E}_{12} - E_{12}^s) - 2\lambda^2}{(\tilde{E}_{12} - \tilde{E}_{11})(\tilde{E}_{12} - \tilde{E}_{22})} \quad ,$$

$$a_{33}^2 = \frac{(\tilde{E}_{22} - E_{11})(\tilde{E}_{22} - E_{12}^s) - 2\lambda^2}{(\tilde{E}_{22} - \tilde{E}_{11})(\tilde{E}_{22} - \tilde{E}_{12})} \quad ;$$

$$a_{11}a_{21} = \sqrt{2}\,\lambda \frac{\tilde{E}_{11} - E_{22}}{(\tilde{E}_{11} - \tilde{E}_{12})(\tilde{E}_{11} - \tilde{E}_{22})} \quad , \quad a_{11}a_{31} = \frac{2\lambda^2}{(\tilde{E}_{11} - \tilde{E}_{12})(\tilde{E}_{11} - \tilde{E}_{22})} \quad ,$$

$$a_{12}a_{22} = \sqrt{2}\,\lambda \frac{\tilde{E}_{12} - E_{22}}{(\tilde{E}_{12} - \tilde{E}_{11})(\tilde{E}_{12} - \tilde{E}_{22})} \quad , \quad a_{12}a_{32} = \frac{2\lambda^2}{(\tilde{E}_{12} - \tilde{E}_{11})(\tilde{E}_{12} - \tilde{E}_{22})} \quad ,$$

$$a_{13}a_{23} = \sqrt{2}\,\lambda \frac{\tilde{E}_{22} - E_{22}}{(\tilde{E}_{22} - \tilde{E}_{11})(\tilde{E}_{22} - \tilde{E}_{12})} \quad ; \quad a_{13}a_{33} = \frac{2\lambda^2}{(\tilde{E}_{22} - \tilde{E}_{11})(\tilde{E}_{22} - \tilde{E}_{12})} \quad ;$$

$$a_{21}a_{31} = \sqrt{2}\,\lambda \frac{\tilde{E}_{11} - E_{11}}{(\tilde{E}_{11} - \tilde{E}_{12})(\tilde{E}_{11} - \tilde{E}_{22})} \quad ,$$

$$a_{22}a_{32} = \sqrt{2}\,\lambda \frac{\tilde{E}_{12} - E_{11}}{(\tilde{E}_{12} - \tilde{E}_{11})(\tilde{E}_{12} - \tilde{E}_{22})} \quad ,$$

$$a_{23}a_{33} = \sqrt{2}\,\lambda \frac{\tilde{E}_{22} - E_{11}}{(\tilde{E}_{22} - \tilde{E}_{11})(\tilde{E}_{22} - \tilde{E}_{12})} \quad .$$

Resonant two-photon transitions $\Phi_{22} \leftrightarrow \Phi_{11}$

between the ground state of the QD and that of the biexciton:

if $\delta > 0$

$$\tilde{E}_{11} = E_0 \ ,$$

$$\tilde{E}_{12} = \frac{E_0 + E_{12}^s}{2} + \frac{1}{2}\Delta ,$$

$$\tilde{E}_{22} = \frac{E_0 + E_{12}^s}{2} - \frac{1}{2}\Delta \ .$$

if $\delta < 0$

$$\tilde{E}_{11} = \frac{E_0 + E_{12}^s}{2} + \frac{1}{2}\Delta \ ,$$

$$\tilde{E}_{12} = \frac{E_0 + E_{12}^s}{2} - \frac{1}{2}\Delta \ ,$$

$$\tilde{E}_{22} = E_0 \ .$$

$\Delta = [(E_0 - E_{12}^s)^2 + 16\lambda^2]^{1/2}$ Rabi frequency.

$$\delta > 0$$

$$a_{11} = \frac{1}{\sqrt{2}} \quad , \quad a_{12} = \frac{2\lambda}{\sqrt{\Delta(\Delta+\delta)}} \quad , \quad a_{13} = \frac{2\lambda}{\sqrt{\Delta(\Delta-\delta)}} \quad ,$$

$$a_{21} = 0 \quad , \quad a_{22} = \frac{1}{\sqrt{2}}\sqrt{\frac{\Delta+\delta}{\Delta}} \quad , \quad a_{23} = -\frac{1}{\sqrt{2}}\sqrt{\frac{\Delta-\delta}{\Delta}} \quad ,$$

$$a_{31} = -\frac{1}{\sqrt{2}} \quad , \quad a_{32} = \frac{2\lambda}{\sqrt{\Delta(\Delta+\delta)}} \quad , \quad a_{33} = \frac{2\lambda}{\sqrt{\Delta(\Delta-\delta)}} \quad .$$

IV. Rabi Flopping of Populations

At the initial time moment t = 0:

- both states with two spin projections $\sigma = \uparrow, \downarrow$ at the lower energy level E_2^0 are occupied by two electrons,
- the upper energy level E_1^0 is empty,

(QD is in the ground state).

At a later time moment $t > 0$:

- Number of the electrons at the upper level

$$\overline{n_1}(t) = \left\langle \Phi_{22} \left| c_1^+(t) c_1(t) \right| \Phi_{22} \right\rangle.$$

- Number of the pairs each of which consists of two electrons with both spin projection $\sigma = \uparrow, \downarrow$ at the same upper energy level

$$n_{11}(t) = \left\langle \Phi_{22} \left| c_{1\uparrow}^+(t) c_{1\uparrow}(t) c_{1\downarrow}^+(t) c_{1\downarrow}(t) \right| \Phi_{22} \right\rangle,$$

biexciton state.

- Number of the pairs each of which consists of one electron at the upper level and another electron at the lower one

$$n_{12}(t) = \left\langle \Phi_{22} \left| c_1^+(t) c_1(t) c_2^+(t) c_2(t) \right| \Phi_{22} \right\rangle.$$

exciton state.

$$\bar{n}_1(t) = 1 - \frac{8\lambda^2}{\Delta(\Delta - \delta)} \cos\frac{\Delta - \delta}{2} t - \frac{8\lambda^2}{\Delta(\Delta + \delta)} \cos\frac{\Delta + \delta}{2} t \,,$$

$$\bar{n}_{11}(t) = \frac{4\lambda^2}{\Delta(\Delta - \delta)} \left(1 - \cos\frac{\Delta - \delta}{2} t\right) + \frac{4\lambda^2}{\Delta(\Delta + \delta)} \left(1 - \cos\frac{\Delta + \delta}{2} t\right) - \frac{2\lambda^2}{\Delta^2} (1 - \cos\Delta t) \,,$$

$$\bar{n}_{12}(t) = \frac{4\lambda^2}{\Delta^2} (1 - \cos\Delta t) \,,$$

$$\bar{n}_1(t) = 2\bar{n}_{11}(t) + \bar{n}_{12}(t) \,.$$

V. Rabi Splitting in Photon Emission Spectra

Effective photon-electron interaction Hamiltonian

$$H_{\text{int}} = \frac{1}{\sqrt{2}} g[(\gamma_x^+ - i\gamma_y^+)c_{2\uparrow}^+ c_{1\uparrow} + (\gamma_x^+ + i\gamma_y^+)c_{2\downarrow}^+ c_{1\downarrow}]e^{i\omega t} + H.c.,$$

Between three two-electron states $\tilde{\Phi}_{11}, \tilde{\Phi}_{12}$ and $\tilde{\Phi}_{22}$

there exist 6 photon emission assisted transitions.

There exist also 3 photon emission assisted transitions

from three states $\tilde{\Phi}_{11}, \tilde{\Phi}_{12}$ and $\tilde{\Phi}_{22}$ to the state Φ_{12}^t

with the energy E_{12}^t .

In the photon emission spectrum there are 9 peaks

with following (relative) intensities and photon energies:

$$I(\tilde{\Phi}_{11} \rightarrow \tilde{\Phi}_{12}) = C\frac{\Delta+\delta}{8\Delta} \quad , \ \hbar\omega = \hbar\omega_0 - \frac{1}{2}(\Delta+\delta) \ ,$$

$$I(\tilde{\Phi}_{12} \rightarrow \tilde{\Phi}_{11}) = C\frac{\lambda^2}{\Delta^2} \quad , \ \hbar\omega = \hbar\omega_0 + \frac{1}{2}(\Delta+\delta) \ ,$$

$$I(\tilde{\Phi}_{12} \rightarrow \tilde{\Phi}_{22}) = C\frac{2\lambda^2\delta^2}{\Delta^3(\Delta+\delta)} \ , \ \hbar\omega = \hbar\omega_0 + \Delta \ ,$$

$$I(\tilde{\Phi}_{22} \rightarrow \tilde{\Phi}_{12}) = C\frac{2\lambda^2\delta^2}{\Delta^3(\Delta-\delta)} \ , \ \hbar\omega = \hbar\omega_0 - \Delta \ ,$$

$$I(\tilde{\Phi}_{11} \to \tilde{\Phi}_{22}) = C\frac{\Delta - \delta}{8\Delta} \ , \ \hbar\omega = \hbar\omega_0 + \frac{1}{2}(\Delta - \delta) \ ,$$

$$I(\tilde{\Phi}_{22} \to \tilde{\Phi}_{11}) = C\frac{\lambda^2}{\Delta^2} \ , \ \hbar\omega = \hbar\omega_0 - \frac{1}{2}(\Delta - \delta) \ ,$$

$$I(\tilde{\Phi}_{11} \to \Phi^t_{12}) = C\frac{1}{4} \ , \ \hbar\omega = \hbar\omega_0 - \delta + E^s_{12} - E^t_{12} \ ,$$

$$I(\tilde{\Phi}_{12} \to \Phi^t_{12}) = C\frac{16\lambda^4}{\Delta^2(\Delta + \delta)^2} \ , \ \hbar\omega = \hbar\omega_0 + \frac{1}{2}(\Delta - \delta) + E^s_{12} - E^t_{12} \ ,$$

$$I(\tilde{\Phi}_{22} \to \Phi^t_{12}) = C\frac{16\lambda^4}{\Delta^2(\Delta - \delta)^2} \ , \ \hbar\omega = \hbar\omega_0 - \frac{1}{2}(\Delta + \delta) + E^s_{12} - E^t_{12}$$

Effect of the electron-phonon interaction:

Some spontaneous photon emission processes are suppressed by the fast exciton relaxation and there appear some new ones,

because

beside of $\tilde{\Phi}_{11}, \tilde{\Phi}_{12}$ and $\tilde{\Phi}_{22}$ there is a separated state Φ^t_{12} .

Suppose that there is some strong non-radiative phonon assisted relaxation mechanism $\Phi^s_{12} \xleftrightarrow{phonon} \Phi^t_{12}$.

If $E^t_{12} > \tilde{E}_{12}$ then Φ^t_{12} plays no role. In the spectrum there are 9 peaks.

If $\tilde{E}_{22} < E^t_{12} < \tilde{E}_{12}$,

$$\tilde{\Phi}_{12} \to \tilde{\Phi}_{11}, \tilde{\Phi}_{12} \to \tilde{\Phi}_{22} \text{ and } \tilde{\Phi}_{12} \to \Phi^t_{12}$$

are suppressed by fast relaxation $\tilde{\Phi}_{12} \xrightarrow{phonon} \Phi^t_{12}$.

If $E^t_{12} < \tilde{E}_{22}\ (< \tilde{E}_{12})$,

$$\tilde{\Phi}_{12} \to \tilde{\Phi}_{11}, \tilde{\Phi}_{12} \to \tilde{\Phi}_{22}, \tilde{\Phi}_{22} \to \tilde{\Phi}_{11}, \tilde{\Phi}_{22} \to \tilde{\Phi}_{12} \text{ and}$$

$\tilde{\Phi}_{12} \to \Phi^t_{12}$, $\tilde{\Phi}_{22} \to \Phi^t_{12}$ are suppressed.

There appear, however, three new photon emission processes

$$\Phi^t_{12} \to \tilde{\Phi}_{11}, \Phi^t_{12} \to \tilde{\Phi}_{12}, \Phi^t_{12} \to \tilde{\Phi}_{22}.$$

VI. Conclusion and Discussions

- The structure of the photon emission spectrum in the Rabi oscillations of three-level system in QD depends on the physical parameters δ and λ , the symmetry of the QD and the polarization of the pumping radiation.
- In the experiments until now $\left|\frac{\lambda}{\delta}\right| << 1$.
- It is very interesting to observe the two-photon Rabi oscillations of biexciton with small binding energy

$$|\delta| \approx |\lambda|$$

in the spontaneous photon emission spectrum.

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