Energy absorption in driven mesoscopic systems

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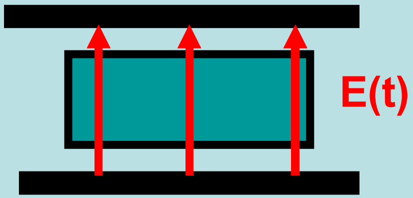
A.Silva, V.E.K. "Multiphoton processes in driven mesoscopic systems" (2006)

D.M.Basko,M.A.Skvortsov,V.E.K. Phys.Rev.Lett.,90, 096801(2003);

D.M.Basko,V.E.K., Phys.Rev.Lett., 93, 056804 (2004)

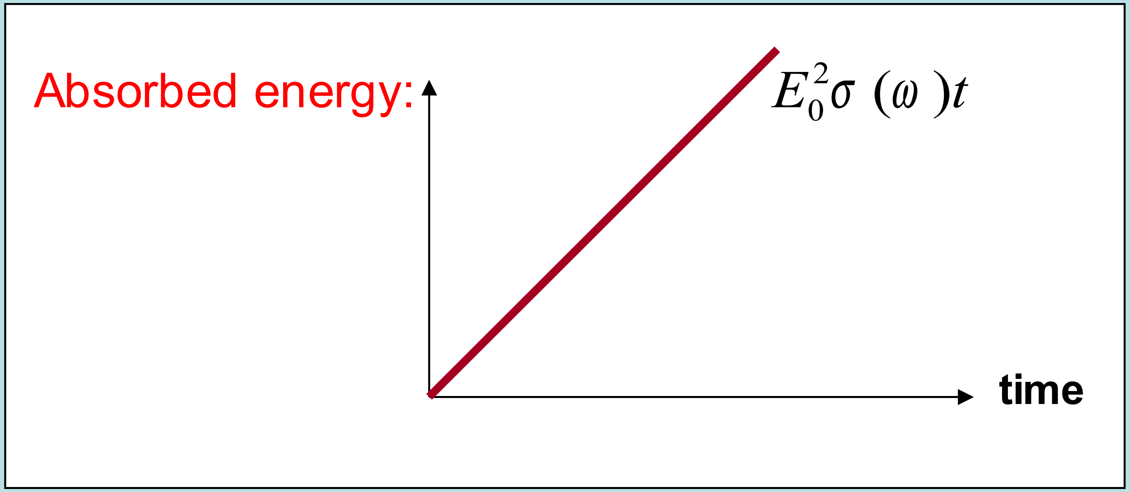
A.Ossipov, V.E.K. Eur.Phys.J. (2005)

Energy absorption from ac field: a textbook picture



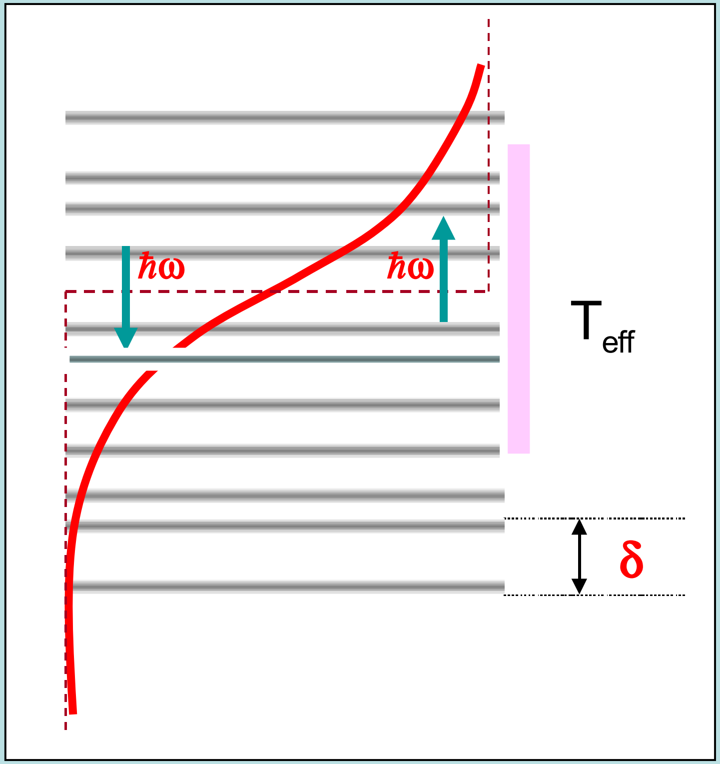
Ac electric field: $E(t) = E_0 \cos(\omega t)$

Absorbed power: $P = E_0^2 \sigma(\omega)$



Absorbed energy $E_0^2 \sigma(\omega) t$ grows linearly in time

Diffusion in the energy space



Total energy:

$$E = const + \int \varepsilon \left[f(\varepsilon) - \theta(\varepsilon) \right] \frac{d\varepsilon}{\delta} \propto \frac{T_{eff}^{2}}{\delta}$$

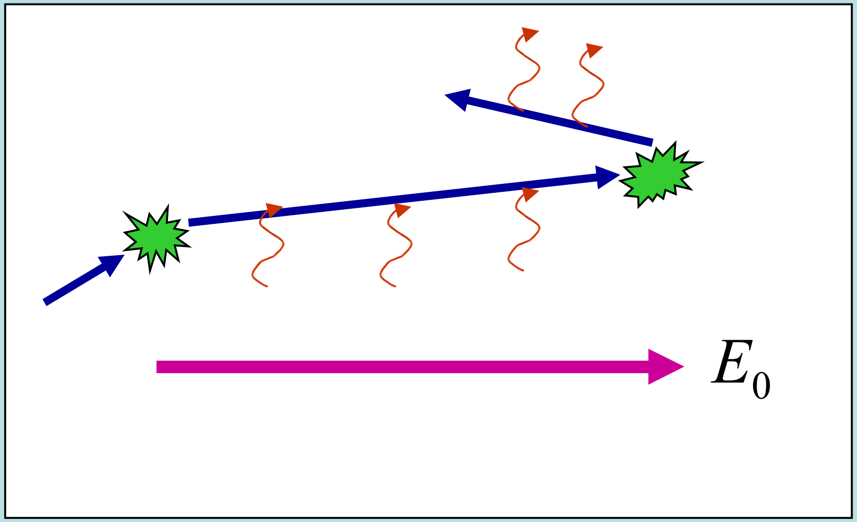
dE/dt = const

$$T_{eff}^{2} = D_E t$$

Diffusion in real and in the energy space

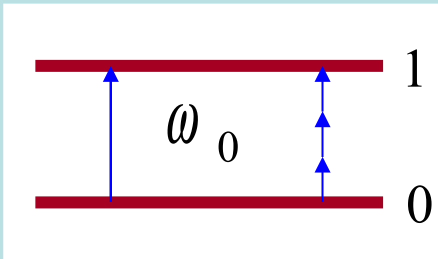
$$D_E = \frac{D[eE_0]^2}{1 + (\omega\tau)^2}$$

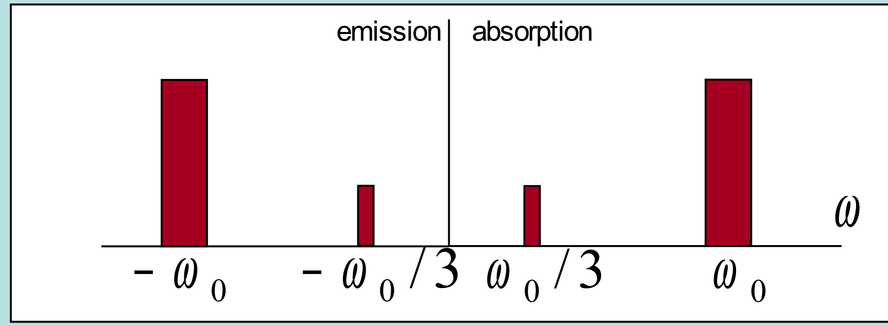
$$D = \ell^2 / \tau d$$



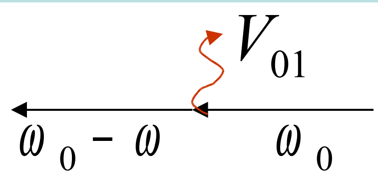
Diffusion in real and in the energy space are related (Joule heat law)

Single- and multi-photon processes



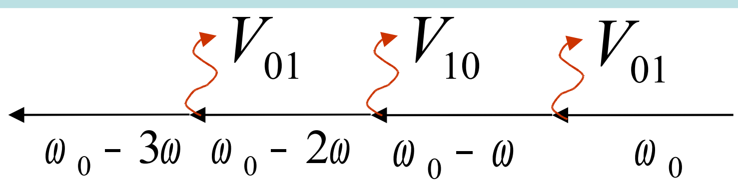


Single-photon emission/absorption



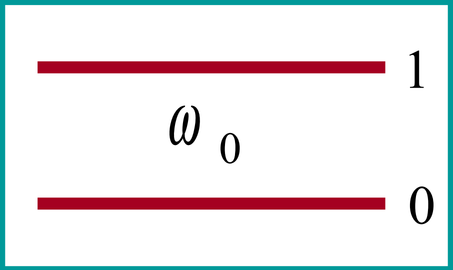
$$\Rightarrow \frac{V_{01}}{\omega_0}\delta(\omega - \omega_0)$$

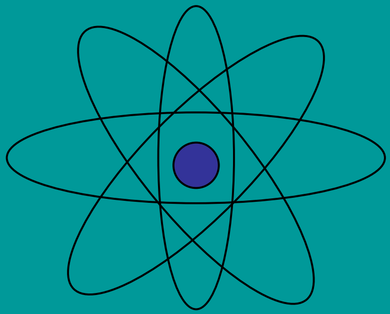
Three-photon emission/absorption

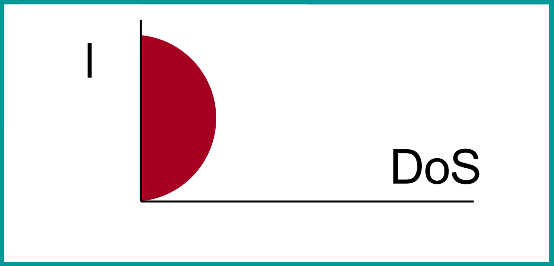


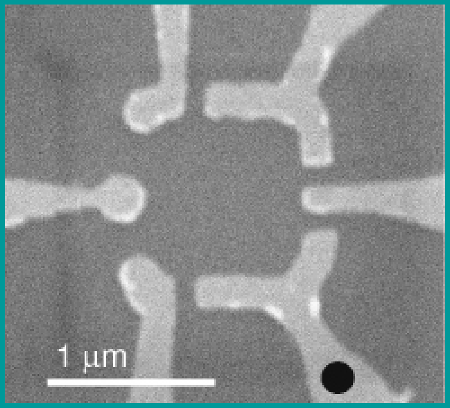
$$\Rightarrow \frac{V_{01}V_{10}V_{01}}{\omega_0(\omega_0 - \omega)(\omega_0 - 2\omega)}\delta(3\omega - \omega_0)$$

An artificial atom-quantum dot: huge number of virtual intermediate states

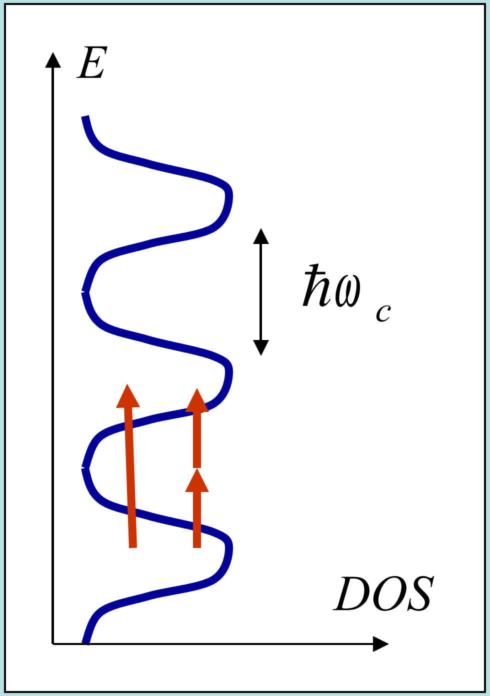








An example: microwave absorption at weak Landau quantization

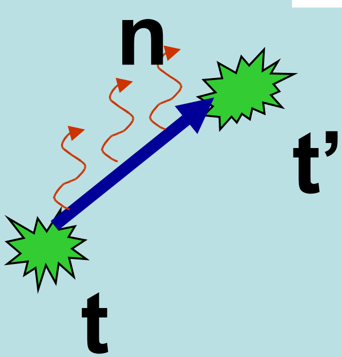


Absorption peak at

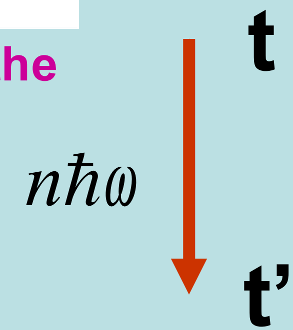
$$\omega = \omega_c / n$$

How to describe? Continuous-Time Random Walk in the energy space

$$f_t(E) = \int_0^t dt' \int_{-\infty}^{+\infty} d\Omega\, \mathcal{P}_\Omega(t,t')\, f_{t'}(E-\Omega).$$



Probability to make a step Ω in the energy space for the time t-t'



$$\mathcal{P}_\Omega(t,t') = \psi(t-t')\, p_\Omega(t,t')$$

$$\psi(t-t') = \frac{1}{\tau}\exp(|t-t'|/\tau)$$

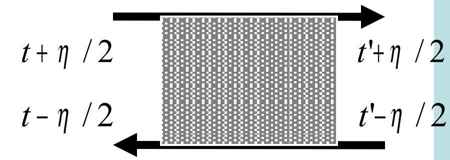
Distribution of waiting times = (correlation times for photon emission/absorption)

$$p_\Omega(t,t') = \sum_{n=-\infty}^{+\infty} \delta(\Omega - n\hbar\omega)\, p_n(t,t')$$

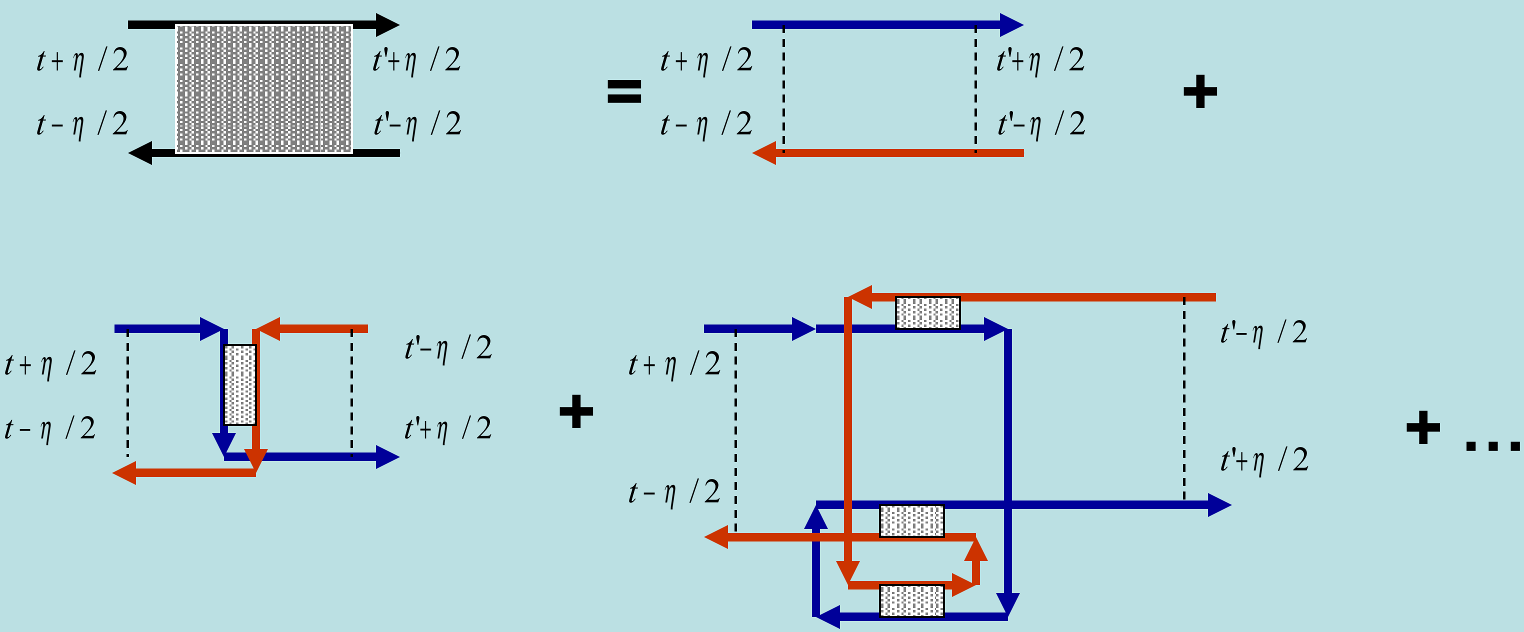
Distribution of step lengths =(number of emitted/ absorbed photons)

How to calculate ?
Language of (interfering) semi-classical trajectories

$$\mathcal{P}_\Omega(t,t') = \int d\eta \, e^{-i\Omega\eta}$$



Semi-classical contribution

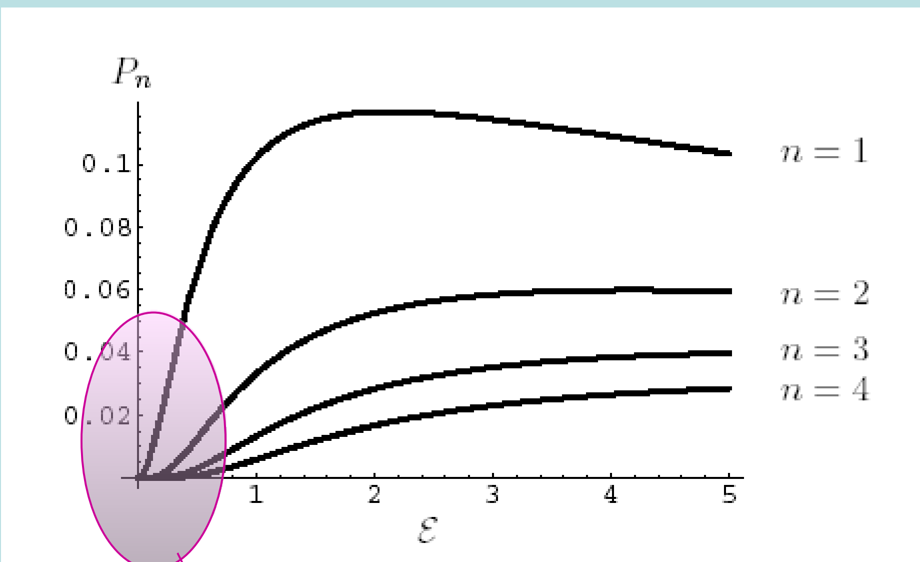


Quantum corrections

Probability of a multi-photon process at short-range disorder

$$P_n = \int_0^t \langle p_n(t,t')\rangle e^{-(t-t')/\tau}\,\frac{dt'}{\tau}$$

$$P_n = A_n E^{2n}\,{}_3F_2(n+1/2,\ n+1/2,\ n+1/2;\ n+3/2,\ 1+2n;\ -4E^2)$$



Here quantization of energy and quantum interference are important

Low intensity: multi-photon proceses are **rare**

$$P_n \approx A_n\,\mathcal{E}^{2n}, \qquad \mathcal{E} = eE_0\ell/(\hbar\omega) \ll 1.$$

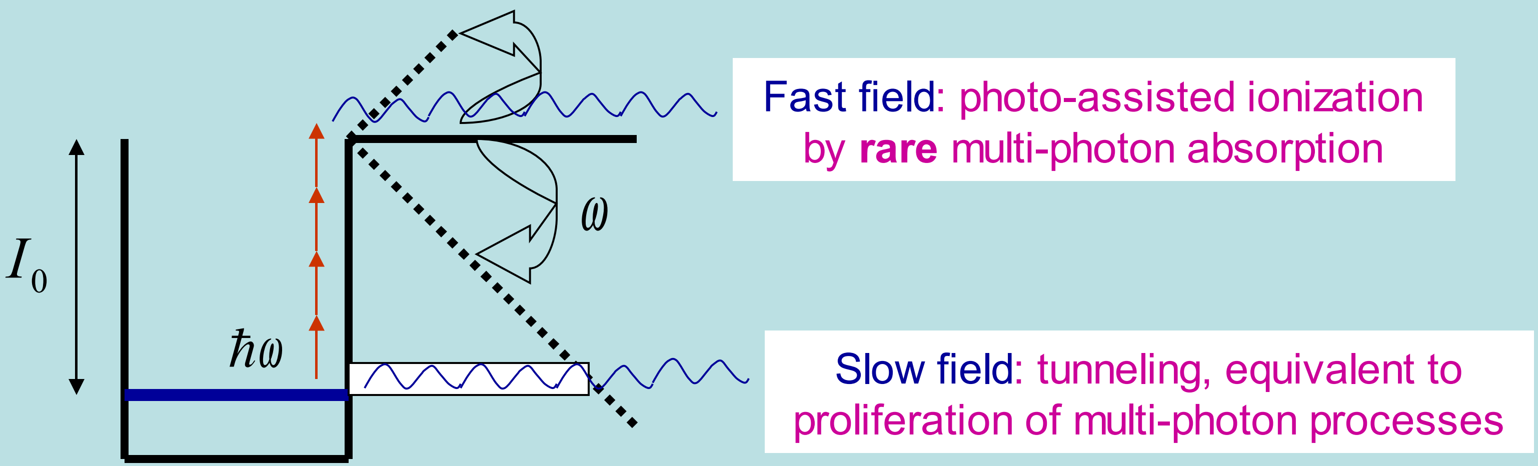
High intensity: proliferation of multi-photon processes

$$P_n \approx \int_{\frac{n}{\mathcal{E}}}^{\infty} \frac{dx}{\pi\mathcal{E}x} K_0(x) \propto \begin{cases} \frac{1}{\mathcal{E}}\ \ln^2(\mathcal{E}/n), & \frac{n}{\mathcal{E}} \ll 1 \\ \exp[-n/\mathcal{E}], & \frac{n}{\mathcal{E}} \gg 1 \end{cases}$$

Follows from classical Drude picture

$$\mathcal{P}_\Omega(t,t') = \psi(t-t')\left\langle \delta\left(\Omega - \int_{t'}^{t} dt'' ev_F \vec{E}(t'')\cdot\hat{n}\right)\right\rangle_{\hat{n}}$$

Connection to the problem of atom ionization



Control parameter: $\left(\omega\tau_{tun}\right)^{-1} = eE_0 a/\hbar\omega\,, \quad a = \hbar/\sqrt{mI_0}\,, \quad \hbar\omega << I_0$

In our problem: $\mathrm{E} = eE_0\ell/\hbar\omega\,, \quad \omega << 1/\tau$

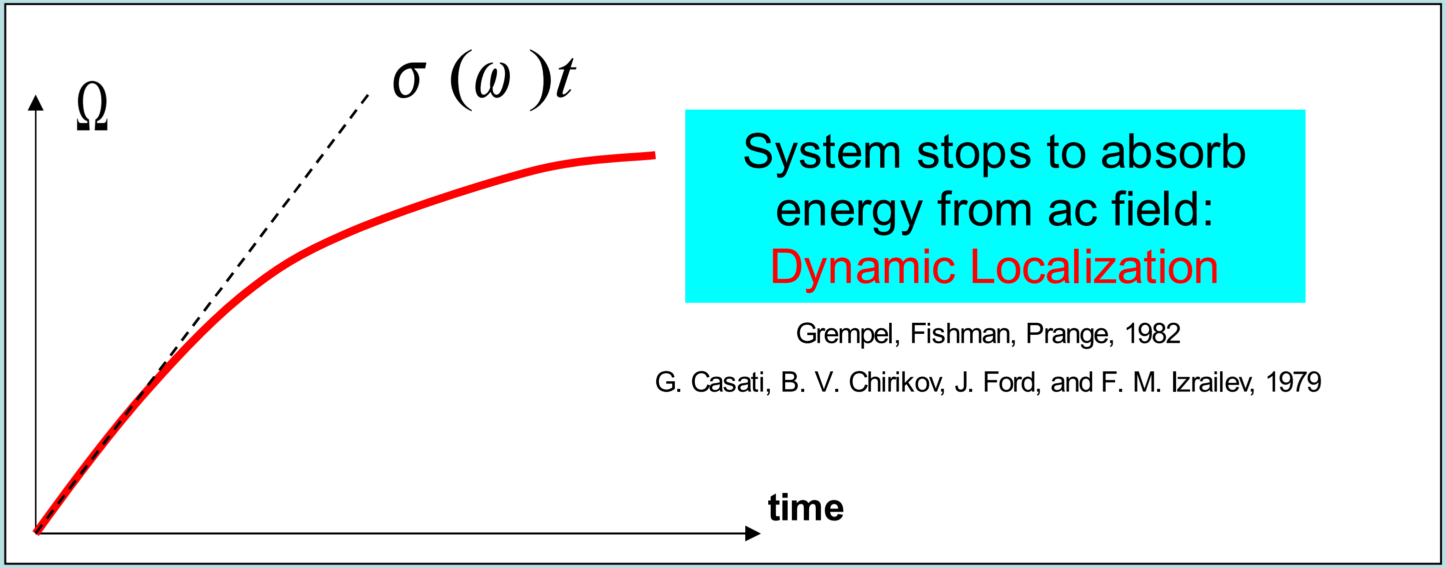
Quantum corrections in isolated mesoscopic systems

Dependence of probability of absorption/emission on the driving time t

$$P_1 = \frac{1}{3}\left(eE_0\ell / \hbar\omega\right)^2\left[1 - \sqrt{\frac{t}{t_{loc}}}\right]$$

Dynamic localization

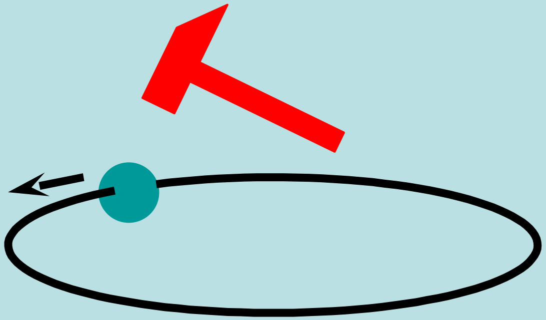
What is dynamic localization?



Kicked rotor:

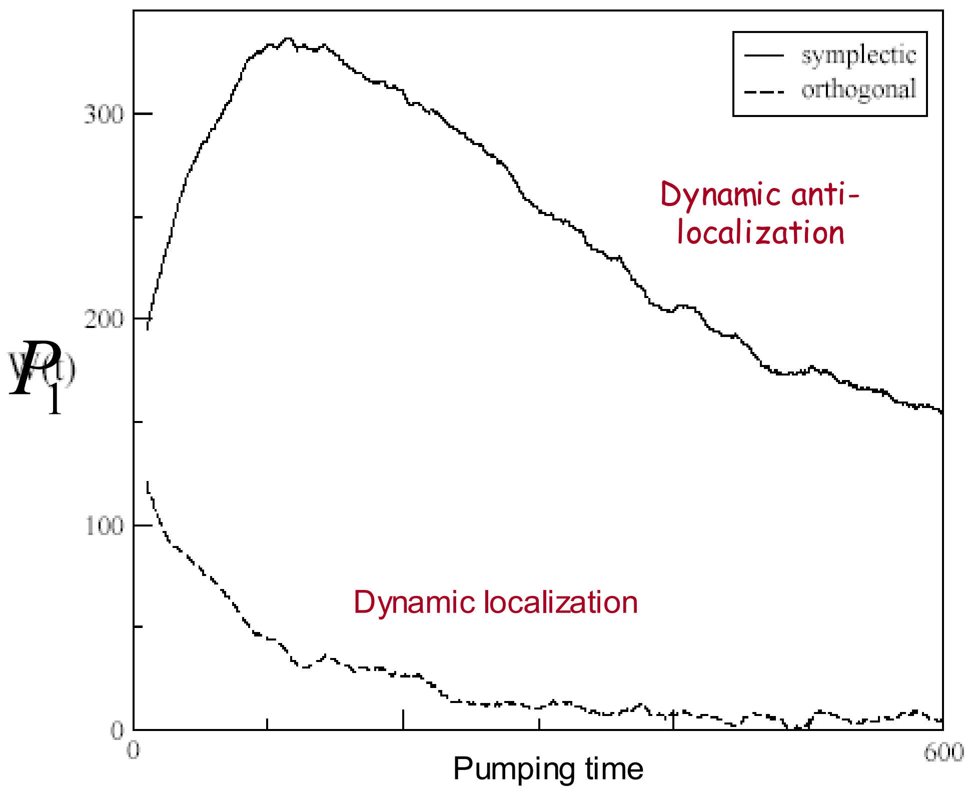
$$\hat{H}(t) = -\frac{\partial^2}{\partial\theta^2} + \cos(\theta)\sum_{n=-\infty}^{\infty}\delta(t-nT)$$

$$\psi_m^{(0)}(\theta) = e^{im\theta}, \quad E_m^{(0)} = m^2$$

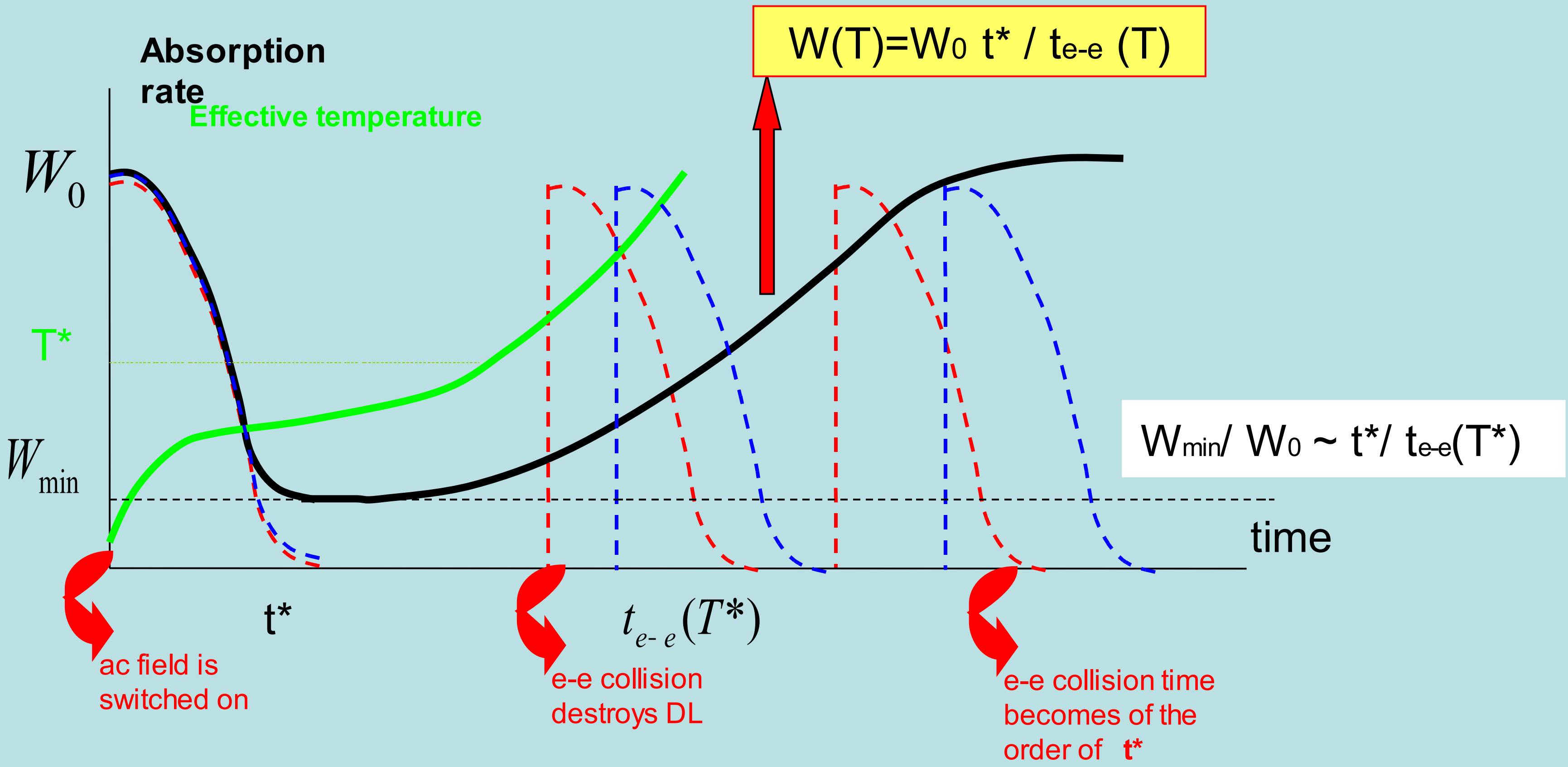


Simulations on QKR

A.Ossipov,
V.E.K
(2005)



Role of electron-electron interaction



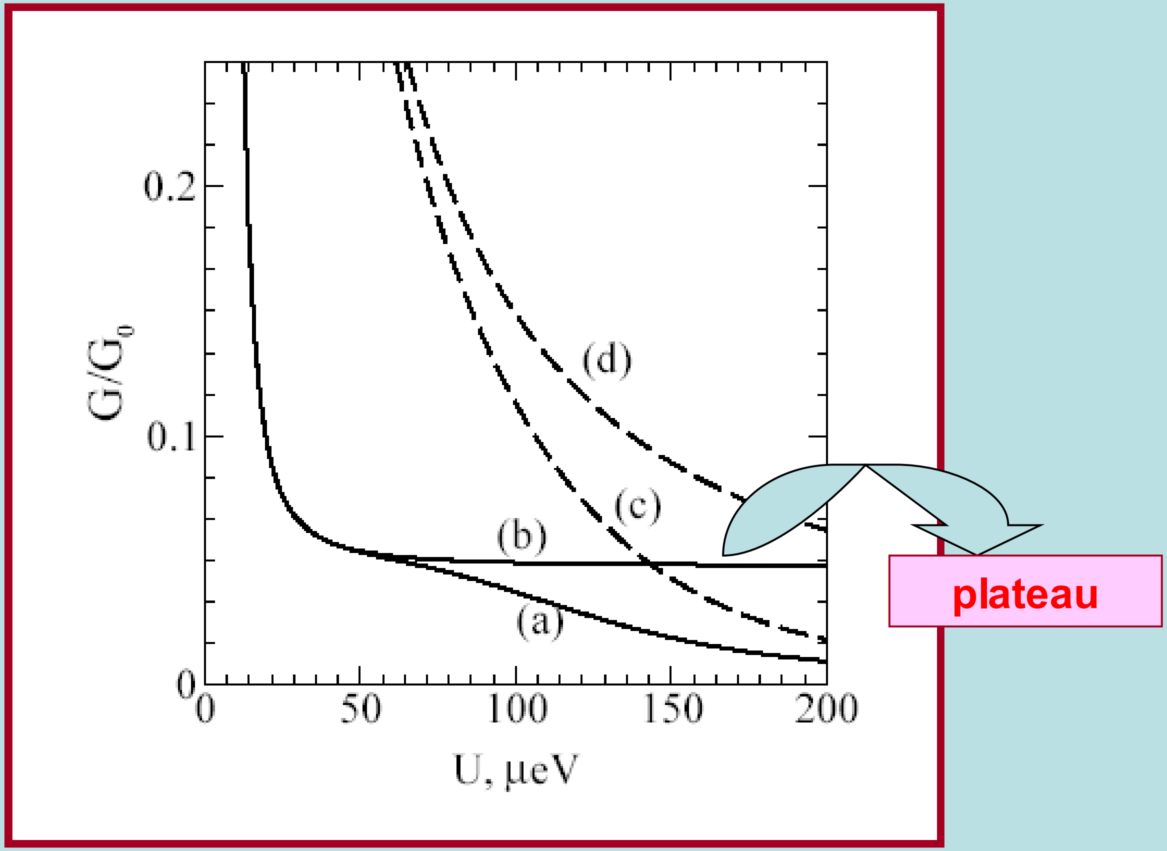
Because of e-e collisions DL is a transient phenomenon!

Conductance vs. gate voltage in the Coulomb blockade regime:

Cooling via hot electron escape: **d). Ohmic regime $W=W_0$; b). Dynamic localization regime.**

Cooling by phonons: **c) Ohmic regime; a).Dynamic localization regime**

Basko, VEK
PRL (2004)



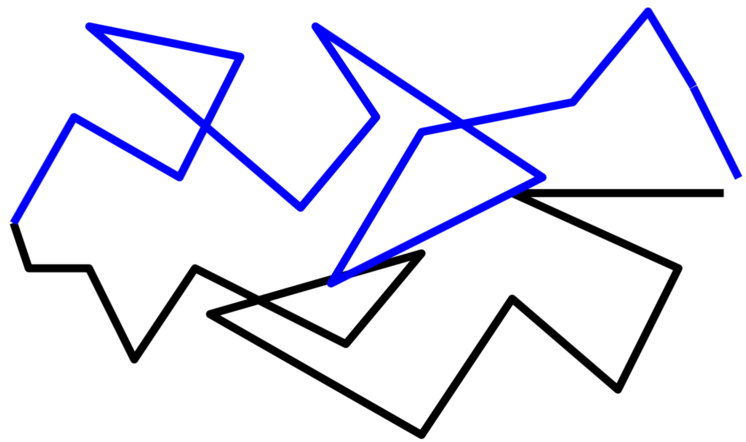
Conclusion

- Diffusion in the energy space is related to diffusion in the real space and is described by the continuous time random walk
- Probability of multiphoton processes is calculated in the quasiclassical approximation
- Quantum corrections bring about dynamic localization
- Electron collisions destroy dynamic localization
- Signature of dynamic localization in the Coulomb blockade regime

Dynamic localization as interference phenomenon

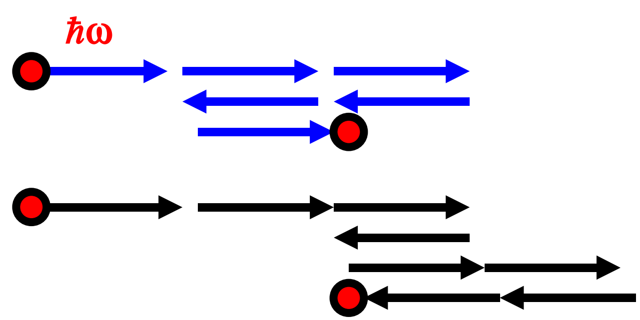
Anderson localization: particle propagation in **real space** is not a Marcovian random walk but rather a result of interference over different trajectories

D(L) -> 0
at large L



$R^2 = D(L)\ t$

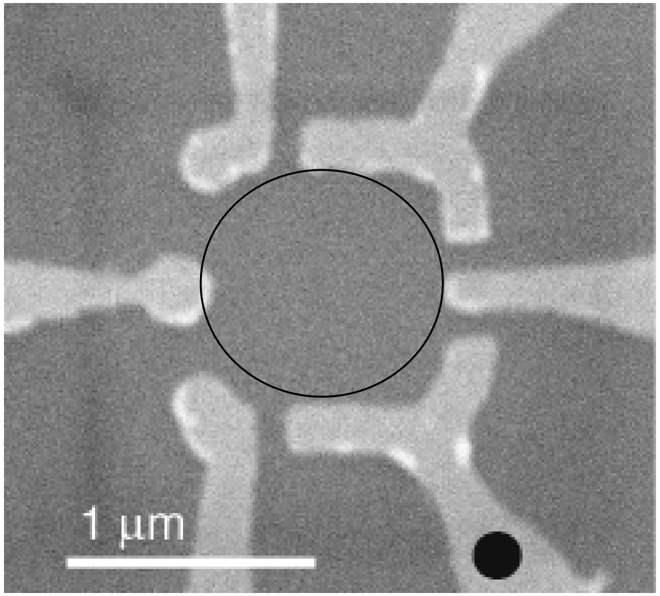
Dynamic localization: particle propagation in **energy space** is also not a Marcovian process of independent absorption and emission of quanta $\hbar\omega$ but the result of interference over different trajectories in the energy space.



D(t) -> 0
at large t

$T^2 = D(t)\ t$

Quantum dot



$$H = H_0 + V\cos(\omega t)$$

H_0, V are random matrices

Is dynamic localization possible?

What to measure?

The model: time-dependent random-matrix theory, closed system, no electron interaction

$$H = H_0 + V\phi(t)$$

$$\mathcal{P}_{H_0} \propto \exp\left[-\frac{\pi^2 \,\mathrm{Tr}\, H_0^2}{4M\delta^2}\right], \quad \mathcal{P}_V \propto \exp\left[-\frac{\pi \,\mathrm{Tr}\, V^2}{4\Gamma\delta}\right],$$

$\phi(t)$ Is an arbitrary function of time obeying

$$\overline{\phi(t)} = 0,$$

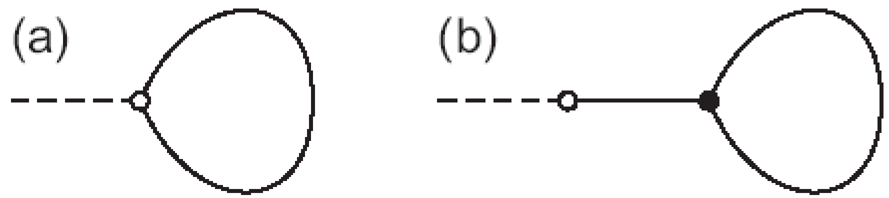
$$\overline{\phi^2(t)} = 1$$

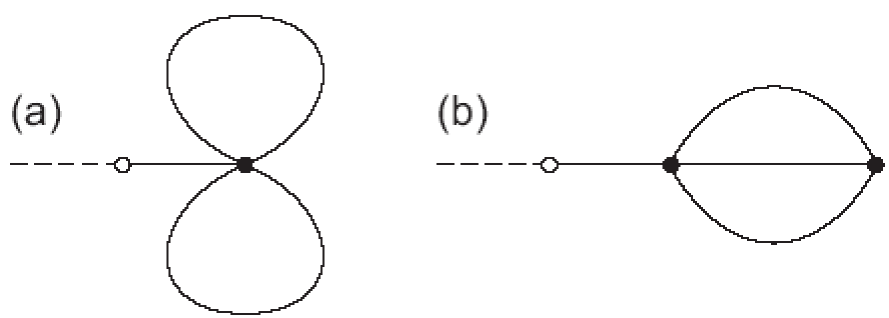
Keldysh Sigma-model

$$S[Q] = \int \left[\frac{\pi i}{2\delta} \operatorname{tr}\{ i\tau_3 \sigma_0 \delta_{tt'} \partial_t Q_{tt'} \} + \frac{\Gamma}{8\delta} [\phi(t) - \phi(t')]^2 \, \operatorname{tr}\{ Q_{tt'} Q_{t't} \} \right] dt \, dt' .$$

$$(Q^2)_{tt'} = \tau_0 \sigma_0 \delta_{tt'} \qquad \tau_2 \sigma_1 Q \sigma_1 \tau_2 = Q^T ;$$

Loop expansion in Keldysh sigma-model





One-loop correction as weak dynamic localization (GOE): $\delta \ll \Gamma$

D.M.Basko, M.A.Skvortsov, V.E.K., 2003

$$D(t) = D_0 + \frac{\Gamma\delta}{\pi}\int_0^t \partial_t\phi(t)\,\partial_t\phi(t-\tau)\,C_{t-\tau/2}(\tau,-\tau)\,d\tau$$

large zero-order

WDL correction ~ - $t^{1/2}$

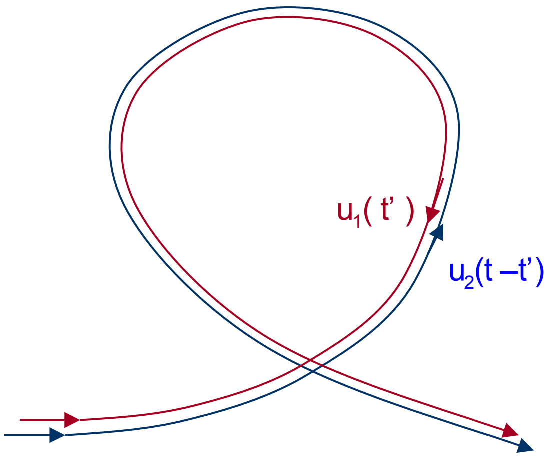
Cooperon keeps track of the quantum interference:

$$C_t(\tau,-\tau) \equiv \exp\left[-\frac{\Gamma}{2}\int_{-\tau}^{\tau}[\phi(t+\xi/2)-\phi(t-\xi/2)]^2\,d\xi\right] \approx \exp[-2\tau\,\Gamma\,\sin^2(\omega t)]$$

dephasing rate for harmonic $\phi(t)=\cos(\omega t)$

Negative WDL correction grows with time if $\Gamma(t)$ has zeros:
no-dephasing points

Physical meaning of no-dephasing points: T_{loop}/T = integer



T_{loop} time of traversing the loop

T period of perturbation

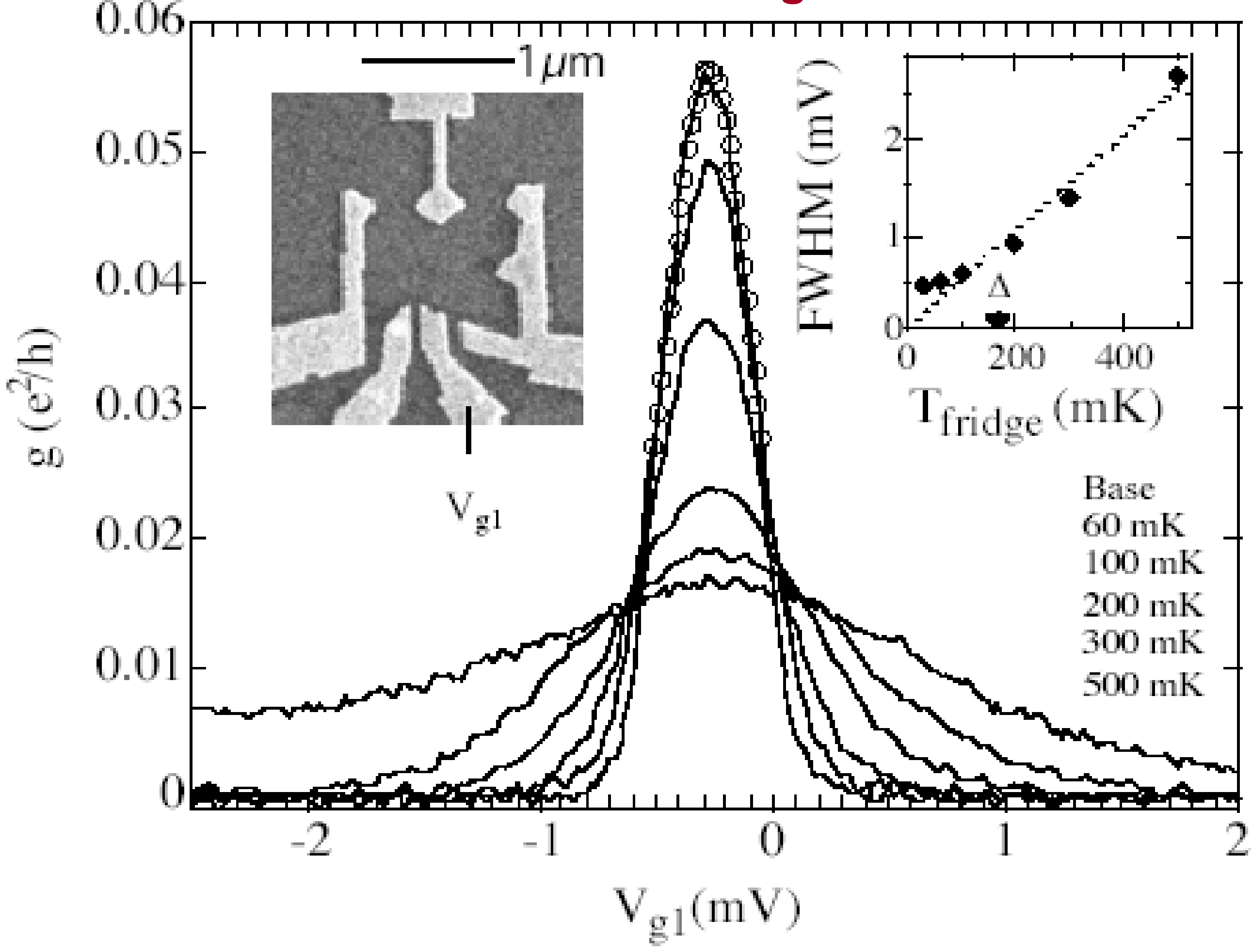
Periodic in time perturbation selects the trajectories with loops by strong dephasing on all trajectories but those with

T_{loop}/T = integer

This selection is the origin of dynamic localization

How to measure effective electron temperature?
Dot weakly connected to reservoirs

Conductance peak in the Coulomb blockade regime



OPEN DOT: balance of heating W_{in} and cooling W_{cool}

$$\frac{W_{in}(T)}{W_0} = t_* \gamma_{def}(T) \qquad \frac{W_{cool}}{T(T - T_0)} = G_0 F\left(\frac{G}{G_0}\right) \approx G$$

Dynamic localization limited by dephasing

e-e interaction: $\gamma_{def}(T) = \delta \left(\frac{T}{E_T}\right)^2$

Analogue of the Wiedemann-Franz law

Low bath temperature: $T_0 << T$

$$G \approx \left(\frac{\omega \Gamma}{E_{Th} \delta}\right)^2 \leq G_0$$

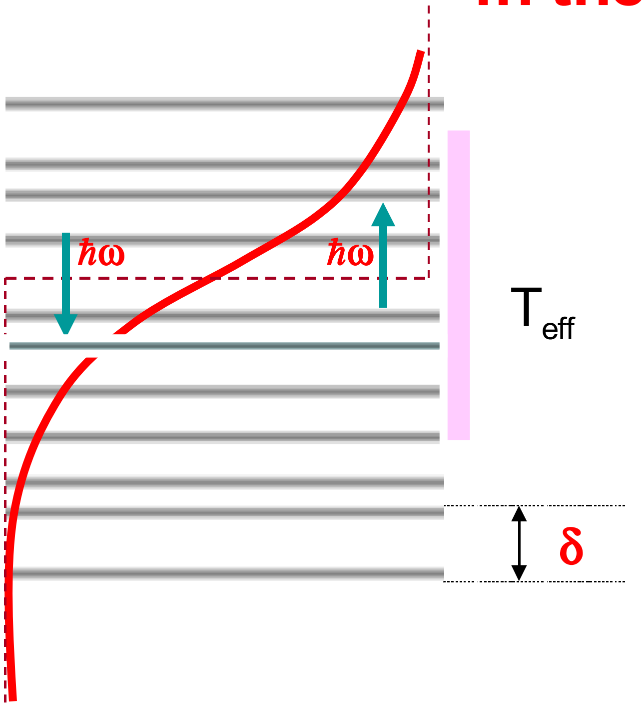
Independent of effective temperature and hence independent of gate voltage:

Plateau in G(U) dependence

Conclusions

- Dynamic localization is possible in complex systems described by random matrix theory and subject to time-periodic perturbation.
- Analytic theory of weak dynamic localization is developed. A qualitative explanation is related with interfering trajectories with loops which traversing time is equal to an integer times the period of perturbation.
- In closed systems an arbitrary weak electron interaction destroys the DL at times of the order of inverse quasi-particle width. This happens because of the combination of dephasing and heating.
- In weakly open quantum dots one may find a regime where the dephasing is not very strong and cooling by electron escape is sufficiently large. In this regime dynamic localization shows up as a plateau on the dependence of conductance vs. gate voltage.

Effective electronic temperature and diffusion in the energy space: isolated dot



Total energy:

$$E = const + \int \varepsilon \left[f(\varepsilon) - \theta(\varepsilon) \right] \frac{d\varepsilon}{\delta} \propto \frac{T_{eff}^2}{\delta}$$

dE/dt = const

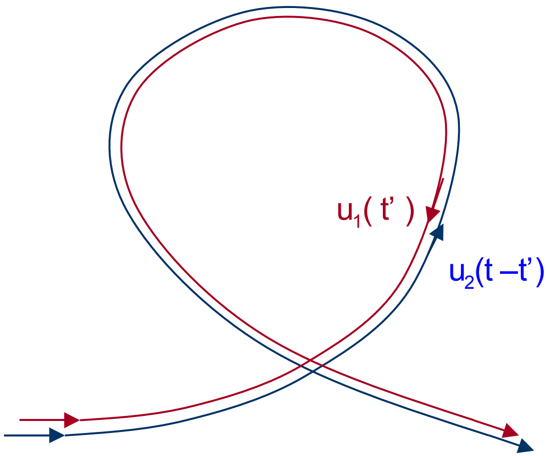
$$T_{eff}^2 = Dt$$

Classical absorption implies diffusion in the energy space with $D = (\hbar\omega)^2 \Gamma \propto ac \quad power$

Dynamic localization implies vanishing the time-dependent diffusion coefficient D(t)=d/dt $T_{eff}^2(t)$ **at large times t**

Physical meaning of no-dephasing points:

t_{loop}/T = integer



$$\delta\varphi = \int_0^t A(t')[u_1(t') - u_2(t')]dt'$$

Loop-traversing time **t**,

External time-dependent vector-potential **A(t')**,

Velocities on time-reversal trajectories $\mathbf{u}_{1,2}$

$u_1(t') = -u_2(t-t')$

For a time-dependence of **A(t')** that is smooth at a scale of velocity correlation time one obtains

$$\langle (\delta\varphi)^2 \rangle \propto \int_{-t/2}^{t/2} d\xi \left[A(t/2+\xi) + A(t/2-\xi)\right]^2$$

For a harmonic **A(t)=sin(ω t)** there are loop-traversing times **t** = 2π κ / ω, k=1,2,…such that **A(t/2 + ξ) = − A(t/2 - ξ)** **for all ξ**

For loops with traversing times t = k T , k=1,2,3… where T is the period of harmonic perturbation no dephasing by external field occurs

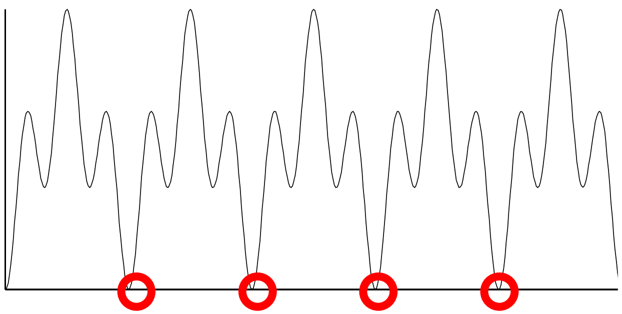
DL for periodic perturbations:

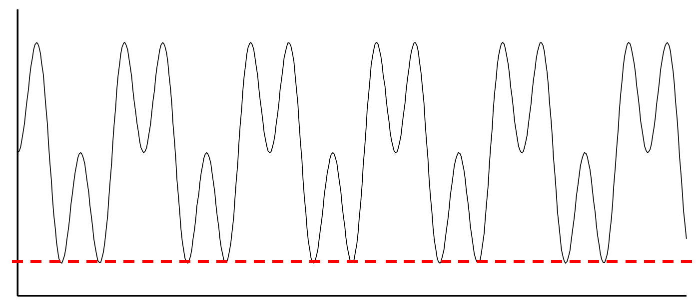
$$\Gamma(t) = \sum_{n=1}^{d} A_n^{\,2} \sin^2(\omega\, n\, t + \varphi_n)$$

X.B.Wang,
V.E.K.,2001

$$\phi(t + t_0) = \phi(-t + t_0)$$

Average dephasing rate $\Gamma(t)$ versus time:





Quasi-1d orthogonal: $\delta D/D \sim -(t / t^*)^{1/2}$

Quasi-1d unitary: $\delta D/D \sim -(t / t^*)$

Monochromatic perturbation: *T*-symmetry always –
a very special case

$$\langle Q_{tt'} \rangle_S = \begin{pmatrix} Q_{tt'}^{R} & Q_{tt'}^{K} \\ 0 & Q_{tt'}^{A} \end{pmatrix} \otimes \tau_3$$

Energy distribution function : $\langle Q^{K}{}_{tt'} \rangle = 2F_{tt'}$

$$F_E(t) = \int d\tau e^{iE\tau} F_{t+\tau/2, t-\tau/2}$$

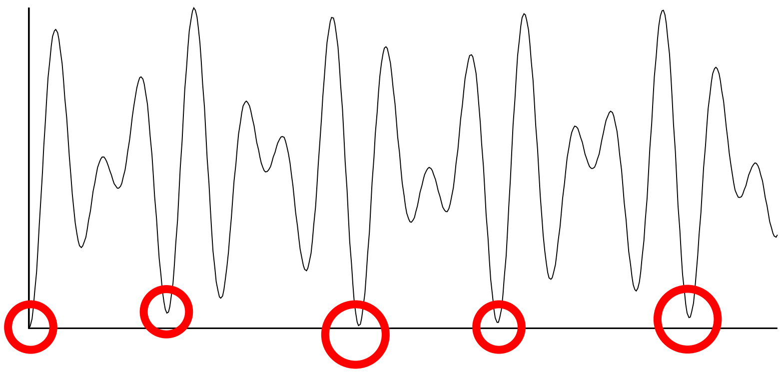
Energy absorption rate:

$$W(t) = \partial_t \langle E(t) \rangle = -\frac{i\pi}{\delta} \lim_{\eta \to 0} \partial_t \partial_\eta F_{t+\eta/2,\, t-\eta/2}$$

Incommensurate periods

$$\Gamma(t) = \sum_{n=1}^{d} A_n^2 \sin^2(\omega_n t + \varphi_n)$$

dephasing rate Γ(t):



Number theory as seen in mesoscopic physics

X.B.Wang, V.E.K. 2001

Almost-no-dephasing points contribute:

$$D(t) - D_0 \sim - D_0 \delta \int_{1/\Gamma}^{t} \frac{dt_1}{\sqrt{(\Gamma t_1)^d}}$$

d-dimensional weak Anderson localization

Basko,Skvortsov, VEK, 2003;

Numerics for kicked rotor: Casati, Guarneri, Shepelyanskii, 1989

Two-loop corrections: weak dynamic localization in driven GUE

$$\delta W(t) = \frac{\Gamma\Delta}{2\pi^2}\lim_{\eta\to 0}\frac{\partial}{\partial\eta}\frac{1}{\eta}\int_0^\infty dx\,dy\,dz$$
$$\times\left(\frac{\partial}{\partial t} - 2\Gamma\varphi_{56}\,\varphi_{78}\right)\varphi_{12}\varphi_{34}$$
$$\times\,\mathcal{D}_{\eta+x+y}\left(t-\frac{x}{2}-\frac{y}{2},\, t-\frac{x}{2}-\frac{y}{2}-z\right)$$
$$\times\,\mathcal{D}_{\eta-x-z}\left(t-\frac{x}{2}-\frac{z}{2},\, t-\frac{x}{2}-\frac{z}{2}-y\right)$$
$$\times\,\mathcal{D}_{\eta+y-z}\left(t-\frac{y}{2}-\frac{z}{2},\, t-\frac{y}{2}-\frac{z}{2}-x\right)$$

$t_{1,2} = t_\pm - x - y - z$, $t_3 = t_+ - z$, $t_4 = t_- - y$ $\quad$ $t_5 = t_+ - x - z$, $t_{6,7} = t_\mp$, and $t_8 = t_- - x - y$

$$\mathcal{D}_\eta(t,t') = \theta(t-t')\,\exp\left\{-\int_{t'}^{t}\Gamma[\varphi(\tau_+)-\varphi(\tau_-)]^2\,d\tau\right\}$$

No dephasing in diffusons at $\tau_+ - \tau_- = \eta = nT$

Dynamic localization in harmonically driven RMT vs Anderson localization in quasi-1d wire

DL GOE: $$\frac{\delta W(t)}{W_0} = -\sqrt{\frac{t}{t^*}}$$

DL GUE: $$\frac{\delta W(t)}{W_0} = -\frac{\pi}{24}\frac{t}{t^*}$$

$$t^* = \frac{\pi^3 \Gamma}{2\delta^2}$$

t_*

AL GOE: $$\frac{\delta\sigma(\omega)}{\sigma_0} = -\frac{1}{\sqrt{-i\omega t_{loc}}}$$

AL GUE: $$\frac{\delta\sigma(\omega)}{\sigma_0} = \frac{1}{6i\omega t_{loc}}$$

$$t_{loc} = (2\pi\nu_1)^2 D_0$$

$$\frac{W(t)}{W_0} = \int_{-\infty}^{+\infty} d\omega \, \frac{e^{-i\omega t}}{(-i\omega + 0)} \left(\frac{\sigma(\omega)}{\sigma_0} \right)$$

Absorption rate in harmonically driven RMT

Conductivity in quasi-1d disordered wire

Keldysh sigma-model: Q(t,t') is a "continuous matrix"

SUSY sigma-model: finite matrix Q

$$S(Q) = \int dx \left[\frac{1}{g} Str(\nabla Q)^2 + i\omega \, Str(\Lambda Q) \right]$$

DL vs AL: strong localization regime

Non-exponential decay of W(t) at large t>>t*

Mott-Berezinskii formula

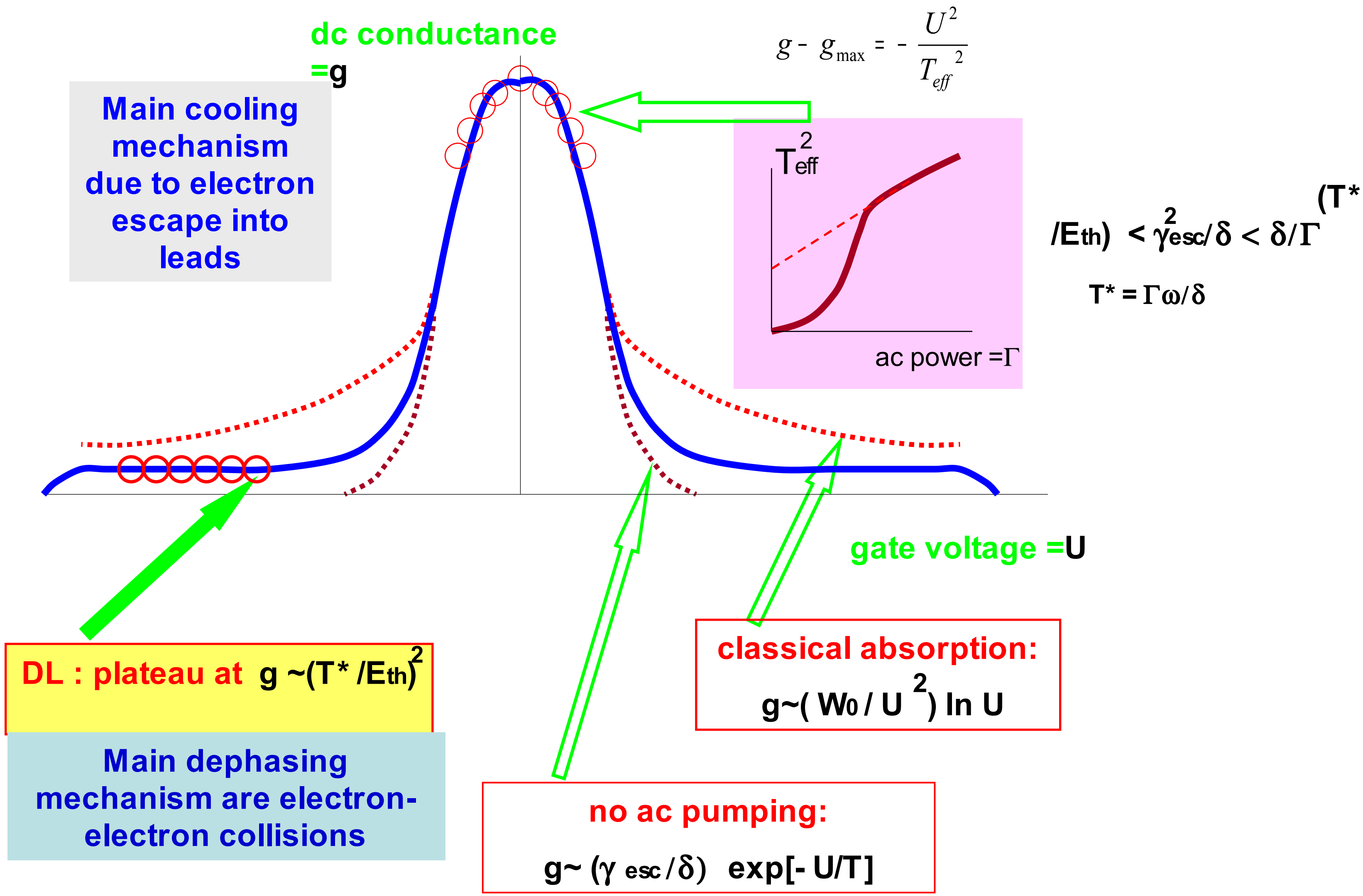
$$\frac{\sigma(\omega)}{\sigma_0} \propto \omega^2 \ln^2 \omega \quad \Rightarrow \quad \frac{W(t)}{W_0} \propto \frac{\ln t}{t^2}$$

DL vs AL: weak anti-localization

$$\frac{\delta W(t)}{W_0} = +\sqrt{\frac{t}{4t^*}}$$

Directly checked

DL and the shape of Coulomb blockade peak under ac pumping



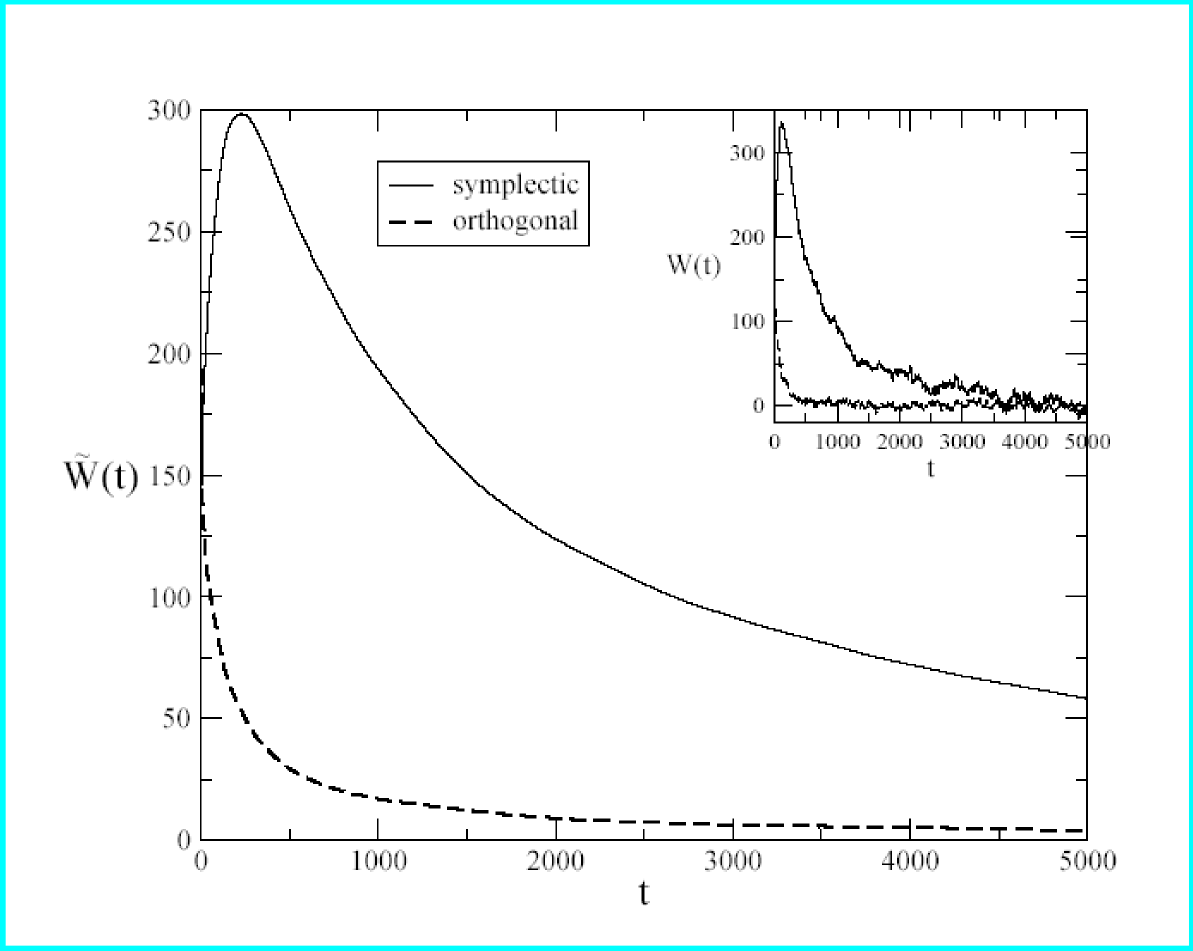
Conclusions

- Chaotic system described by RMT exhibits dynamic localization if perturbation is nearly harmonic $A_n < n^{-3/2}$

- Physical origin of DL are trajectories with loop-traversing time t close to k T

Absorption rate in SU(2) quantum kicked rotor

$$\hat{H} = \frac{1}{2}\hat{L}^2 + K[\cos\alpha \ \cos\beta \ \cos\theta + \cos\alpha \ \sin\beta \ \sin 2\theta \quad \hat{\sigma}_x + \sin\alpha \ \sin\theta \quad \hat{\sigma}_z]\delta(t)$$



Strong DL is destroyed by weak dephasing

$$D(T_{eff}) = (t^*/\tau_\varphi)\, D_0 \propto T_{eff}^2$$

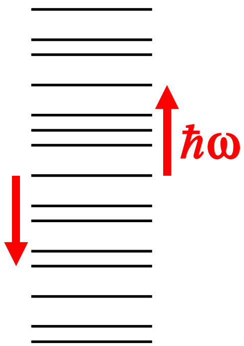
Fermi Golden rule approximation

No analogue of exponentially small diffusion for strong Anderson localization in the regime of hopping conductivity

? Effect of localization in the Fock space **?**

How to understand DL?

Resonance transitions under a periodic perturbation

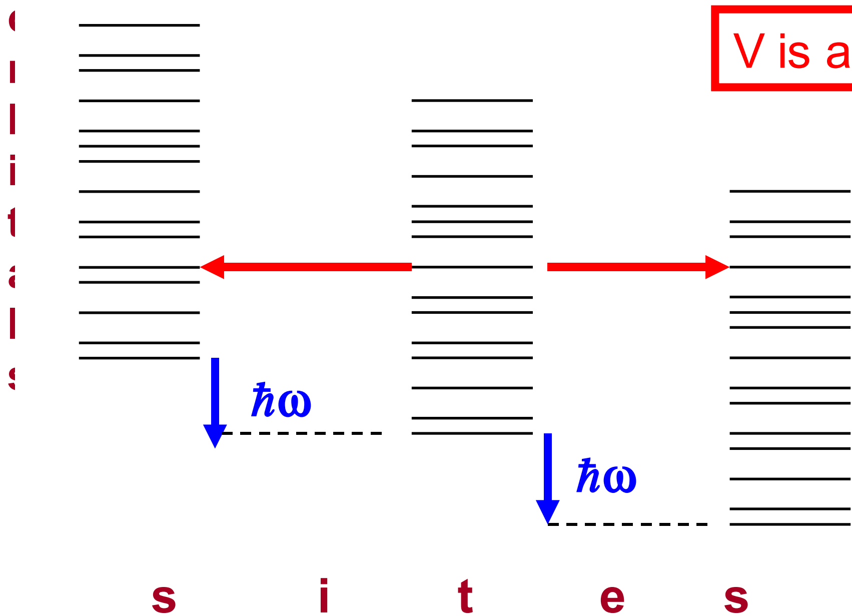


$$H = H_0 + V\cos(\omega t)$$

Resonance transitions under a stationary perturbation

$$H = H_0 + V$$

V is a random matrix



N orbital Anderson model: Anderson localization !

Periodic perturbation: no-dephasing points

$$\phi(t) = \sum_{n=1}^{\infty} A_n \cos(n\omega t - \varphi_n) \qquad |A_n| < \frac{1}{n^{3/2}}$$

$$D_0 = \frac{\Gamma \omega^2}{2} \sum_n n^2 A_n^2$$

$\Gamma = V^2/\delta$ is proportional to the ac power

$$C_t(\tau, -\tau) = \exp[-\Gamma(t)\tau] \approx \exp\left[-\tau \sum_{n=1}^{\infty} \Gamma A_n^2 \sin^2(n\omega t - \varphi_n)\right]$$

If $\varphi_n = n\varphi$ the dephasing rate $\Gamma(t)$ can vanish at $t_k = \dfrac{\varphi + k\pi}{\omega}$

No-dephasing points t_k give a negative correction to D(t) that grows with time:

$$D(t) - D_0 \sim -\omega^2 \delta \sqrt{\Gamma t}$$

At **t~(1/δ) (Γ/δ)=t*** the correction is of the order of **D_0**

How to compute WDL for d-incommensurate harmonics

$$\phi(t) = \sum_{n=1}^{d} \cos(\omega_n t - \varphi_n) \qquad \Gamma(t) = \Gamma \sum_{n=1}^{d} \sin^2(\omega_n t + \varphi_n)$$

$$C_t(\tau,-\tau) = \exp[-\Gamma(t)\tau] \qquad e^{-z\cos\theta} = \sum_{k=-\infty}^{+\infty} (-1)^k I_k(z) e^{ik\theta}$$

$$\frac{D(t)}{D_0} = 1 - \frac{\delta}{\pi\Gamma}\int_0^{\Gamma t} e^{-zd} \sum_{k_n=-\infty}^{\infty} \mathrm{I}_{k_1}(z)\ \mathrm{I}_{k_2}(z)\ldots\mathrm{I}_{k_{d-1}}(z)\left(\frac{d\mathrm{I}_{k_d}(z)}{dz}\right)\exp\left[i\sum_{n=1}^{d} k_n\left(\omega_n(2t - z/\Gamma) + \tilde{\varphi}_n\right)\right]dz$$

Averaging over time interval $1/\omega \ll \Delta t \ll t$

$$\sum_{n=1}^{d} k_n\omega_n = 0$$

For incommensurate frequencies is only satisfied if all k_n=0

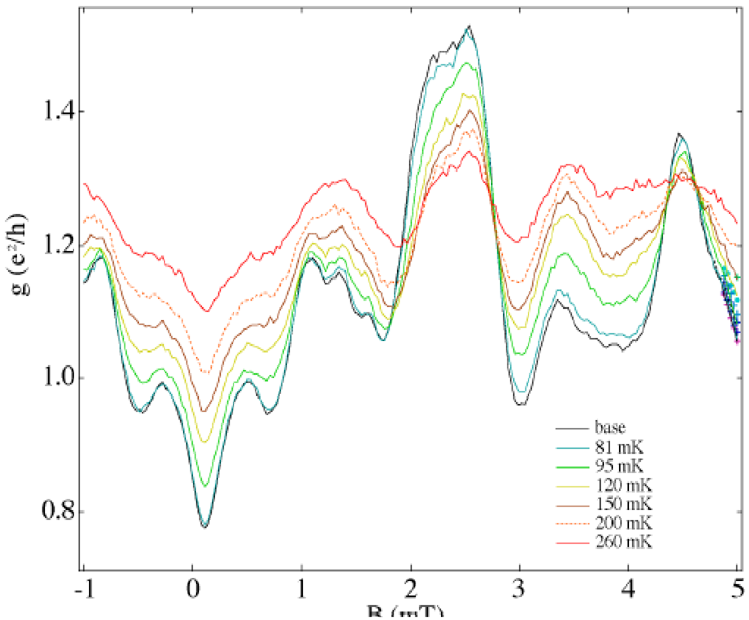
$$\frac{D(t)}{D_0} = 1 - \frac{\delta}{\pi\Gamma}\int_0^{\Gamma t} e^{-zd}\,[\mathrm{I}_0(z)]^{d-1}\left(\frac{d\mathrm{I}_0(z)}{dz}\right)dz$$

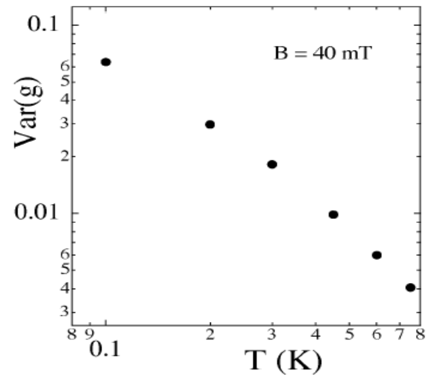
$$I_0(z) \approx dI_0/dz \approx e^z/\sqrt{2\pi z}$$

at $z \sim \Gamma t \gg 1$

$$\frac{D(t)}{D_0} = 1 - \frac{\delta}{\pi\Gamma}\int_0^{\Gamma t} \frac{dz}{[2\pi z]^{d/2}}$$

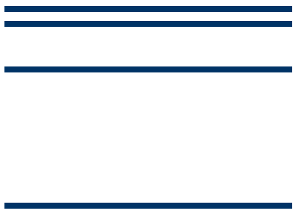
Mesoscopic conductance fluctuations in an open quantum dot: the variance decrease with increasing temperature





Experimental realizations: WHAT TO MEASURE?

Rydberg levels in the microwave field



J.E.Bayfield,P.M.Koch,1974

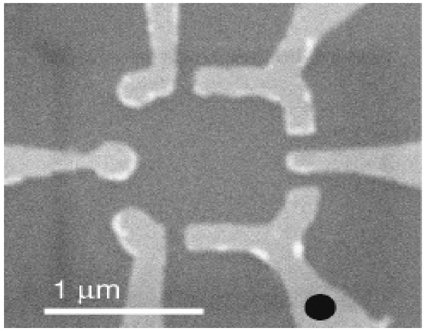
Ionization rate

Two-level system (atom) in a resonant standing wave

F.L.Moore et al, 1994

Momentum distribution

Quantum dot under ac pumping



Di Carlo, Marcus, Harris, 2003

Effective temperature

Functional representation: the Keldysh sigma-model

$$S[Q] = \int \left[\frac{i\pi}{2\delta} \mathrm{Tr}\{ i\tau_3 \sigma_0 \delta_{tt'} \partial_t Q_{tt'} \} + \right.$$
$$\left. + \frac{\Gamma}{4} [\phi(t) - \phi(t')]^2 \; \mathrm{Tr}\{ Q_{tt'} Q_{t't} \} \right] dt\, dt'.$$

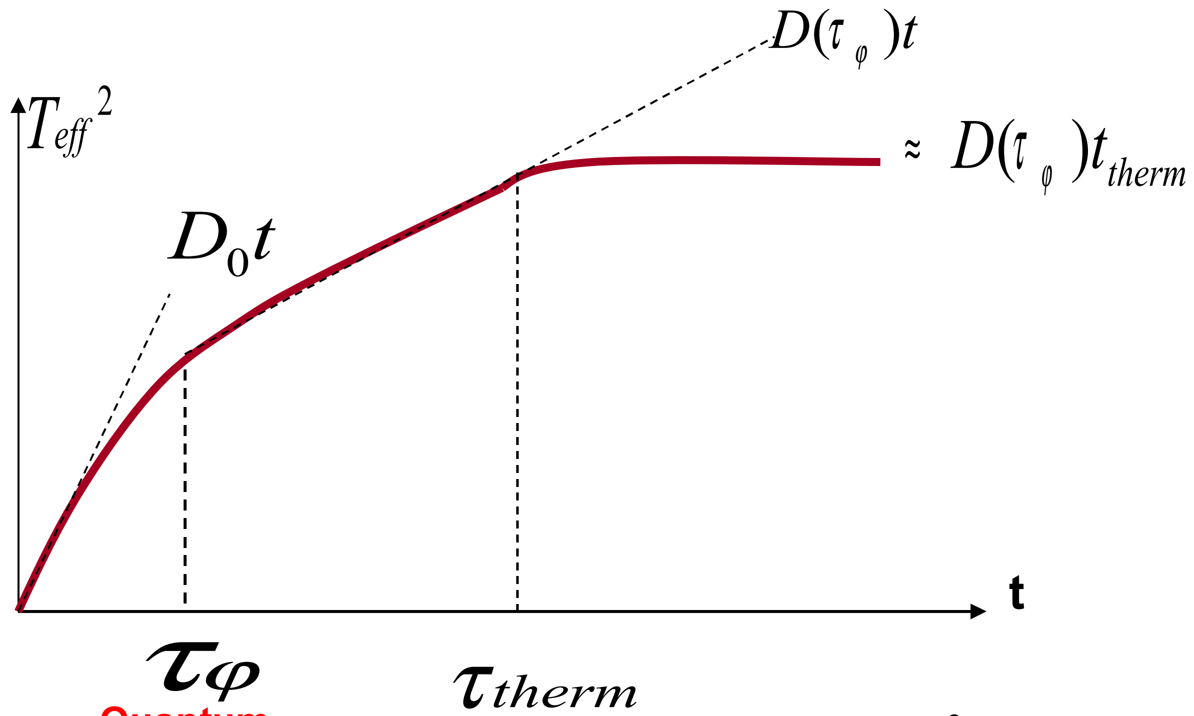
$$\int dt'' Q_{tt''} Q_{t''t'} = \delta_{tt'}$$

Q is **4Mx4M** time-dependent matrix in the direct product of the particle-whole space and Keldysh space; **τ** and **σ** are Pauli matrices in these spaces

Energy distribution function:

$$f(E,t) = \frac{1}{2}\left(1 - \int \langle Q_{11}^{12} \rangle_{t+\tau/2,\, t-\tau/2} \; e^{iE\tau} d\tau \right)$$

Role of dephasing and thermalization



Quantum mechanics is switched off

Steady state is reached

$$\frac{1}{\tau_\varphi} \approx \delta \frac{T_{eff}^2}{E_T^2}, t_{loc} \approx \frac{\Gamma}{\delta^2}, T_{eff} \approx \frac{\omega\Gamma}{\delta}$$

$$\frac{t_{loc}}{\tau_\varphi} \approx \frac{\Gamma^3}{\delta^3}\frac{\omega^2}{E_T^2}$$

Localization time could be smaller than the dephasing time if the Thouless energy is large enough

Bistability and switching

Dephasing time itself depends on effective temperature

$$\tau_{\varphi}^{-1} \propto T_{eff}^{2}$$

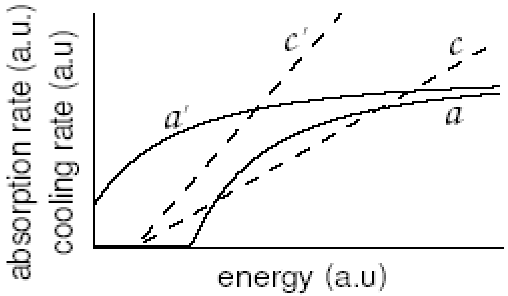


FIG. 1: A schematic plot of possible dependencies of the absorption (solid lines a and a') and the cooling rate (dashed lines c and c') on E from Eq. (4).

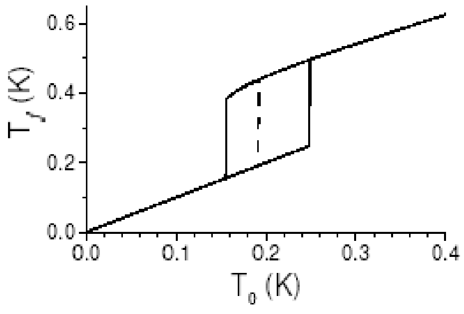


FIG. 3: The final temperature T_f of the system at $t \to \infty$ as a function of the reservoir temperature T_0 for the kinetic and the quasistationary experiments described in the text (dashed and solid lines, respectively). Other parameters are the same as for Fig. 2.

A glance at the reality

GaAs dot:

- size $L \sim 1\,\mu\mathrm{m}$
- mean level spacing $\delta \sim 1\,\mu\mathrm{eV}$
- Thouless energy $E_{Th} \sim 100 - 1000\,\mu\mathrm{eV}$
- dephasing time $t_{\varphi} \sim 1 - 10\,\mathrm{ns}$

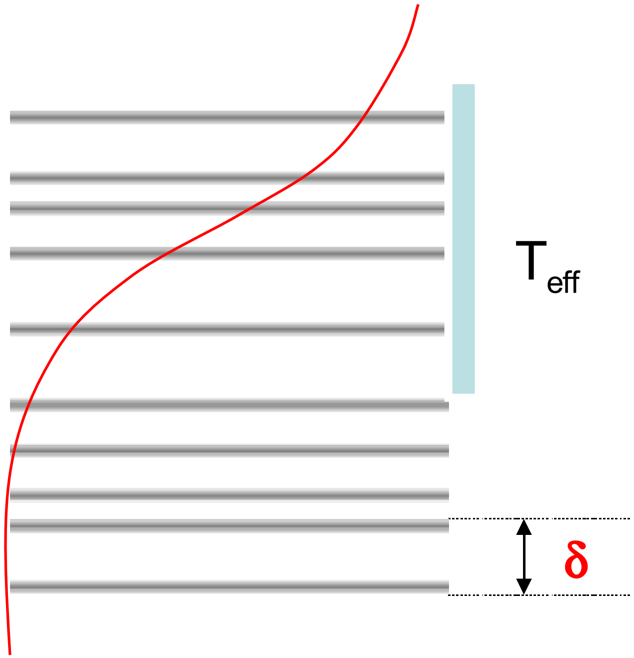
Microwave field:

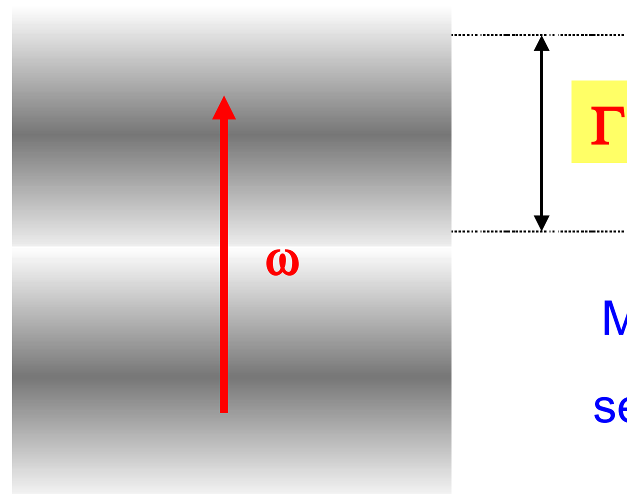
- V~ several μeV (field ~ several 100 V/m)
- $\hbar\omega \sim 10 - 100\,\mu\mathrm{eV}\,(\sim 10^{10}\ \mathrm{Hz})$

Dynamic localization:

- $t_{loc} \sim 10\,\mathrm{ns},\ E_{loc} \sim \sqrt{D t_{loc}} \sim 100 - 1000\,\mu\mathrm{eV} \sim 1 - 10\,\mathrm{K}$

Infinite system: the case of large perturbation





$\Gamma = V^2/\delta > \delta$

Spectrum is continuous to the first approximation: resonance does not play any role

Many levels involved ⇨ semiclassical picture

Diffusion in the energy space:

$l \rightarrow \omega$; $\tau \rightarrow 1/\Gamma$

$D = l^2/\tau \rightarrow \Gamma\,\omega^2$

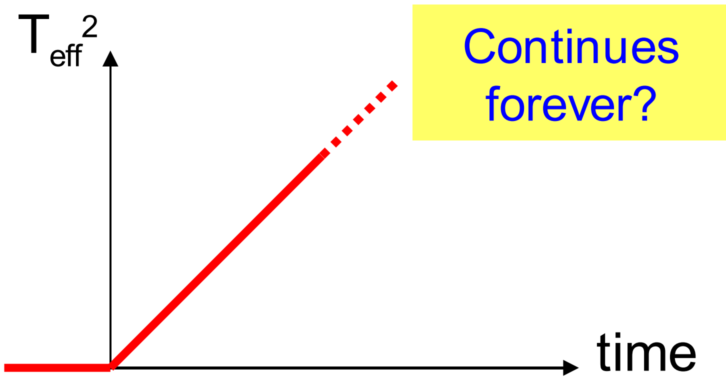
Total energy:

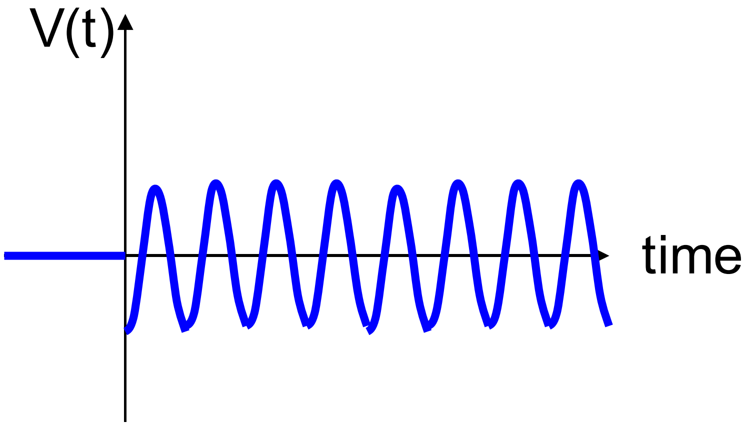
$$E = const + \int \varepsilon\,[f(\varepsilon) - \theta(\varepsilon)]\frac{d\varepsilon}{\delta} \propto \frac{T_{eff}^{\;2}}{\delta}$$

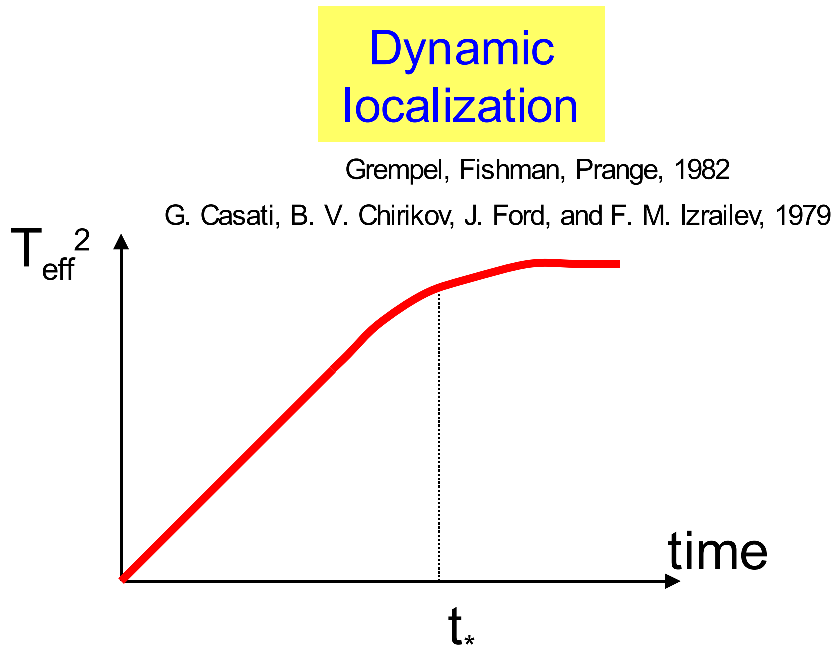
$dE/dt = D/\delta = const$

$T^2_{eff} = D\,t$

Dynamic localization





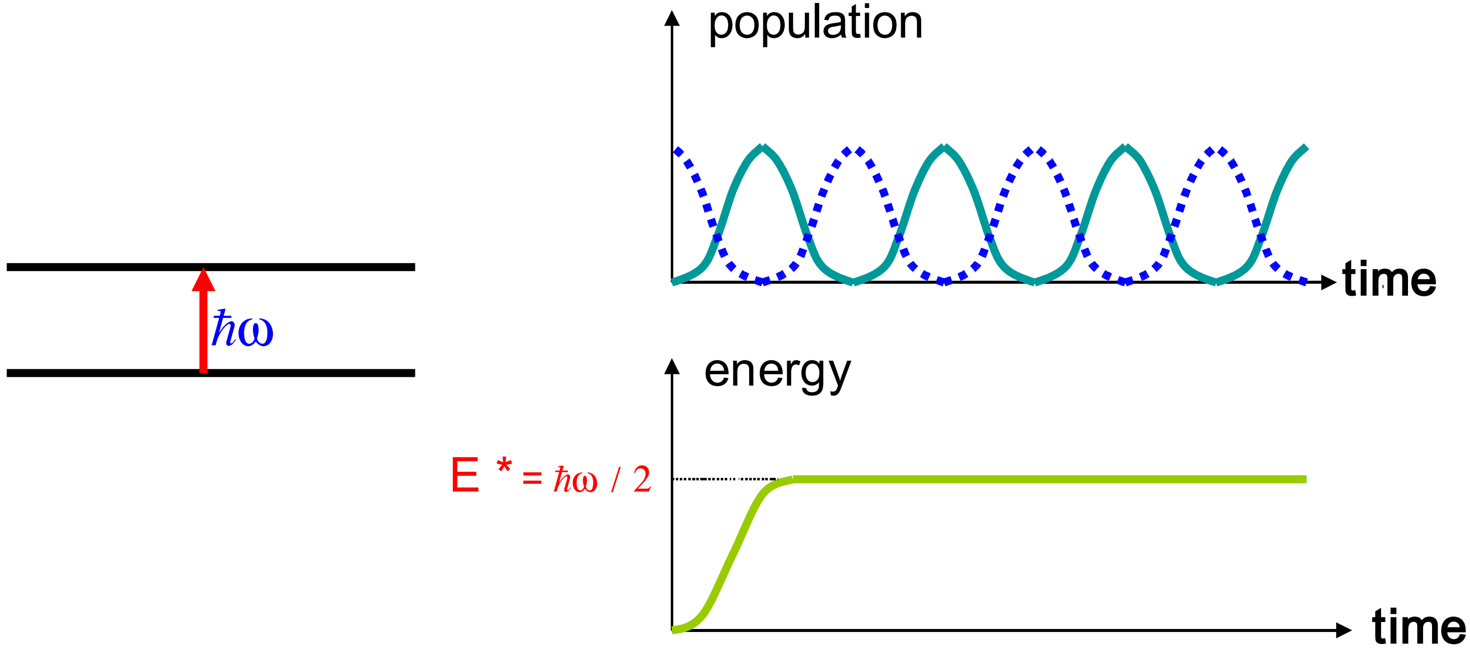


$t_* \rightarrow$ infinity at $\delta \rightarrow$ **0**

memory of quantization

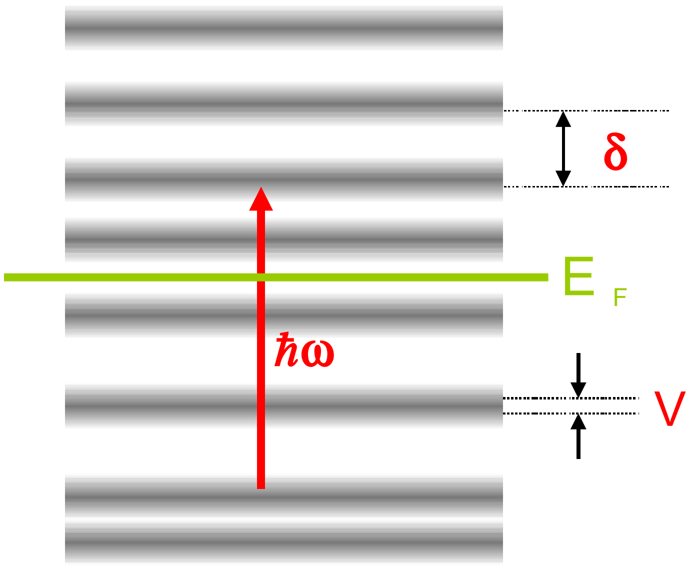
How general is this effect? How to describe analytically?

Localization in a two-level system



Population oscillates (Rabi oscillations), energy saturates

Infinite system: the case of small perturbation



$$H = H_0 + V\cos(\omega t)$$

$V \ll \delta$

Probability to fall in resonance **(V/δ)**

Number of allowed active initial levels **(ω/δ)**

Saturation energy of a resonant pair **$\hbar\omega$**

For $t > t^* = \hbar/\delta$ the total energy of an infinite system saturates at

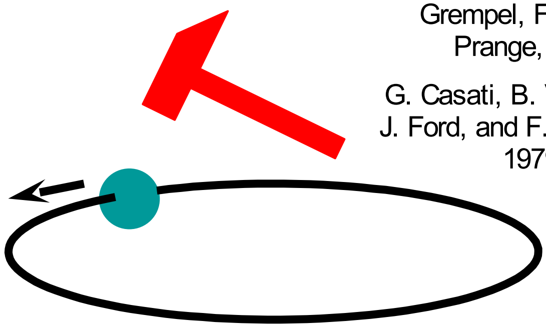
$$E^* \sim V\,(\omega/\delta)^2$$

Wilkinson, Austin, 1992

Kicked rotor:

$$\hat{H}(t) = -\frac{\partial^2}{\partial\theta^2} + V(\theta)\sum_{n=-\infty}^{\infty}\delta(t-nT)$$

$$\psi_m^{(0)}(\theta) = e^{im\theta}, \quad E_m^{(0)} = m^2$$



Grempel, Fishman, Prange, 1982

G. Casati, B. V. Chirikov, J. Ford, and F. M. Izrailev, 1979

$V(\theta) = \cos(\theta)$

$V_{mm'} = V(\delta_{m',m+1} + \delta_{m',m-1})$

only neighboring states are connected by perturbation

$$f(t) \propto \sum_{n=-\infty}^{+\infty}\delta(t-nT) = \sum_{n=-\infty}^{+\infty}\cos(\omega nt)$$

all harmonics have the same amplitude

Can the results on dynamic localization in this system be extended to a generic chaotic system (random matrix)???

No analytic results for $\Gamma >> \delta$

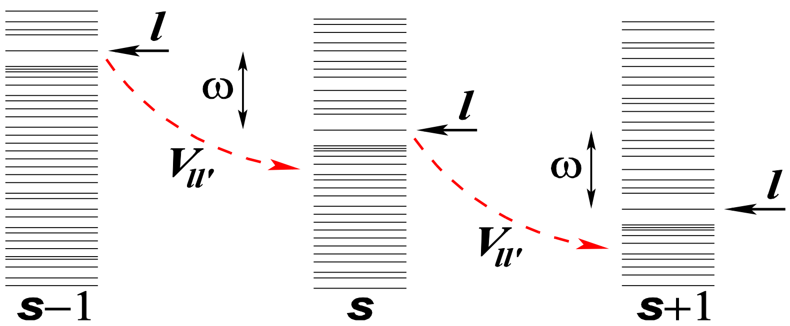
Random matrix with almost harmonic perturbation

$$H = H_0 + Vf(t);$$

GOE GOE

$$f(t) = \sum_n A_n \cos(\omega_n t + \varphi_n)$$

Few harmonics are relevant: $A_n < \frac{1}{n^{3/2}}$



Few sites are connected

DYNAMIC LOCALIZATION IS POSSIBLE

Weak dynamic localization

Basko, Skvortsov, V.E.K, 2003

$$\partial_t E = W^{(0)} + \frac{\Gamma}{2\pi^2}\int_0^t \partial_t f(t)\,\partial_t f(t-\xi)\,\mathcal{C}_{t-\xi/2}(\xi,-\xi)\,d\xi$$

Classical diffusion

Quantum interference correction

Altshuler, Aronov, Khmelnitskii, 1982

Yudson, Kanzieper, V.E.K. 2001

Vavilov, Aleiner, 2001

diffuson: $\mathcal{D}_\eta(t,t') = \theta(t-t')\exp\left[-\Gamma\int_{t'}^{t}[f(t_1+\eta/2)-f(t_1-\eta/2)]^2\,dt_1\right]$ (γ_d)

cooperon: $\mathcal{C}_t(\eta,\eta') = \theta(\eta-\eta')\exp\left[-\frac{\Gamma}{2}\int_{\eta'}^{\eta}[f(t+t_1/2)-f(t-t_1/2)]^2\,d\eta_1\right]$ (γ_c)

Dephasing factors

Harmonic perturbation with high frequency: $\omega \gg \Gamma \gg \delta$

$\gamma_c(t-\xi/2) = \sin^2[\omega(t-\xi/2)]$

No-dephasing windows near

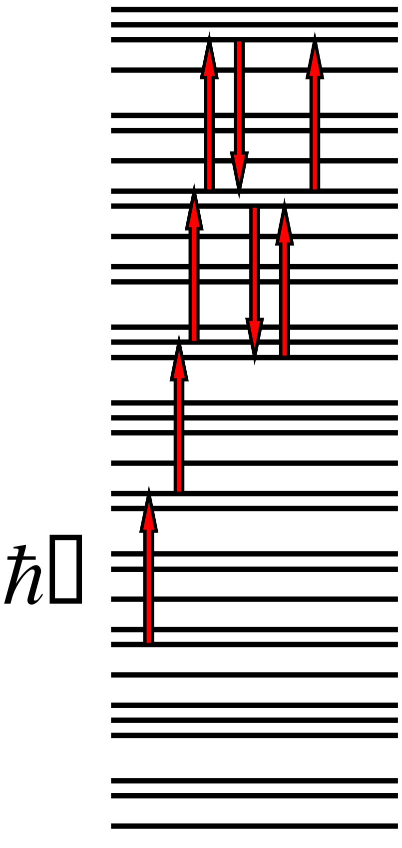
$\omega\,(t-\xi_n/2) = \pi n$

make the main contribution

$$W = W^{(0)}\left[1-\sqrt{\frac{t}{t_*}}\right]$$

$$t_* = \pi^3\Gamma/(2\delta^2)$$

What everybody knows…



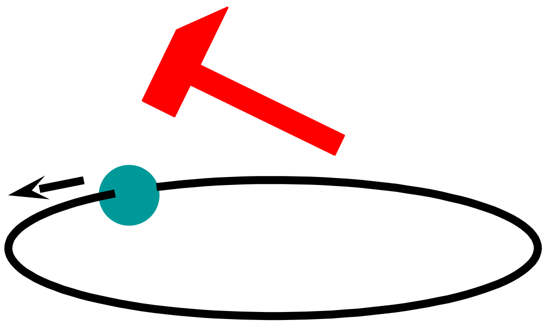
- $\hat{H} = \hat{H}_0 + \hat{V}\cos\omega t$
- (Quasi)continuous spectrum
- Absorption and emission of quanta $\hbar$□ random walk up and down
- Diffusive evolution of the electron distribution function

What some people know…

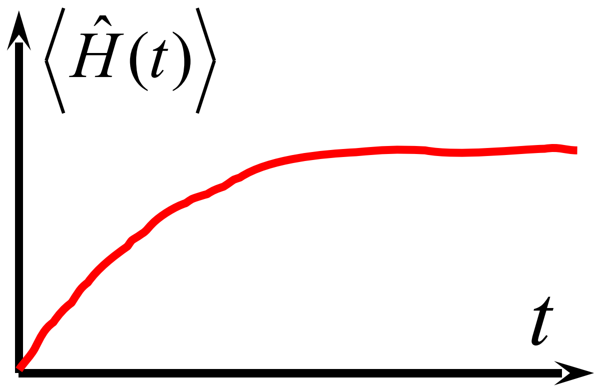
Kicked rotor:

$$\hat{H}(t) = -\frac{\partial^2}{\partial \theta^2} + V(\theta)\sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$\psi_m^{(0)}(\theta) = e^{im\theta}, \quad E_m^{(0)} = m^2$$



Dynamic localization in the energy space:
after some time the rotor stops absorbing



(G. Casati, B. V. Chirikov, J. Ford, and F. M. Izrailev, 1979)

Historical developments

1. Quantum interference – analogous to the Anderson localization (Fishman, Grempel, and Prange, 1982)
2. Incommensurate periods T_1, T_2, T_3 – 3D localization (Casati, Guarneri, Shepelyansky, 1989)
3. Particle in a box: just $\psi(0) = \psi(2\pi) = 0$ instead of the periodic $\psi(0) = \psi(2\pi)$ – no localization (Hu, Li, Liu, Gu, 1999)
4. 996)

What do these observations mean and how general are they?

Spatial localization

Quantum correction to the diffusion coefficient of electrons in disorder

$$D - D_0 \sim -\frac{D_0}{\nu}\int_0^{1/l}\frac{d^d\vec{k}}{D_0k^2 + 1/t_\varphi}$$

mean free path

density of states

dephasing time

Change variables $D_0k^2 = 1/t$:

$$D - D_0 \sim -\frac{1}{\nu}\int_\tau^{t_\varphi}\frac{D_0dt}{(D_0t)^{d/2}}$$

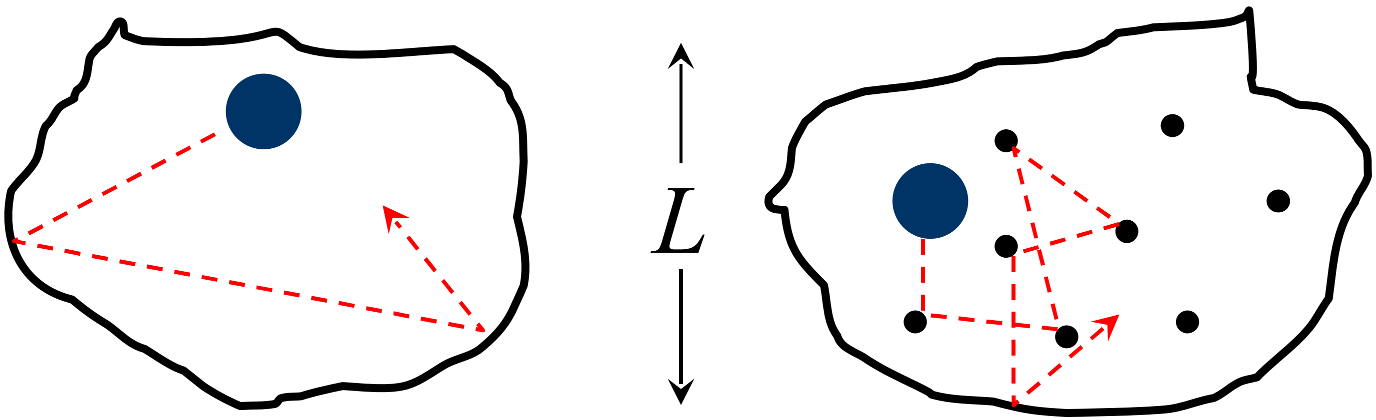
Localization: $d = 1: \quad L_{loc} \sim \nu D_0 \sim l$

$d = 2: \quad L_{loc} \sim l\exp(\nu D_0)$ (?)

$d \geq 3:$ no localization in weak disorder

Chaotic systems

Ballistic systems: Diffusive systems:



$\tau_{erg} = L / v_F$ ergodic time $\tau_{erg} = L^2 / D$

RMT is valid at low energies:

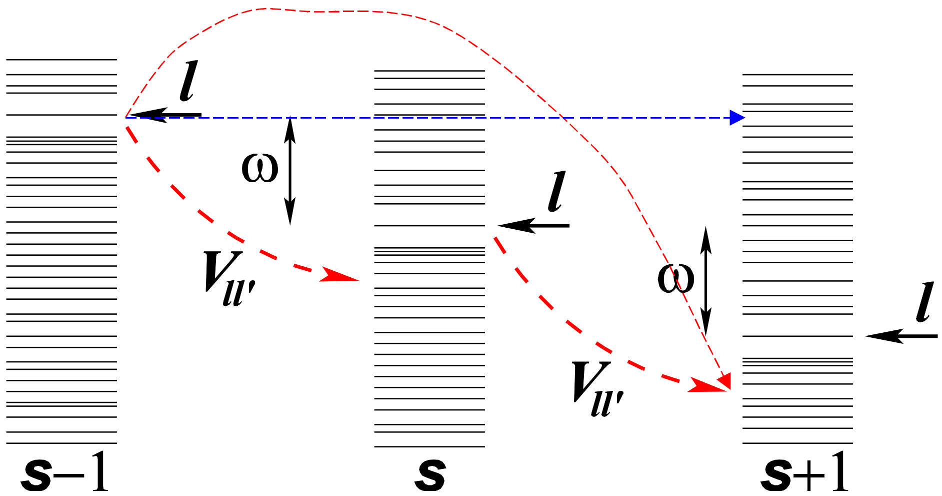
$E << E_{Th} = \hbar / \tau_{erg}$ (Thouless energy)

Perturbation by periodic δ–function (kicks)

$$H = H_0 + Vf(t);$$

$$f(t) = \sum_{n=-\infty}^{+\infty} \delta(t - 2\pi n/\omega) = \sum_{n=-\infty}^{+\infty} \cos(\omega n t)$$

All sites are directly connected



Kicked rotor :

$$V_{mm'} = V(\delta_{m',m+1} + \delta_{m',m-1})$$

only neighboring orbitals are connected: remote sites are out of resonance → dynamic localization

Kicked random matrix :

$$\langle V_{ll'}^2 \rangle = const$$

NO DYNAMIC LOCALIZATION

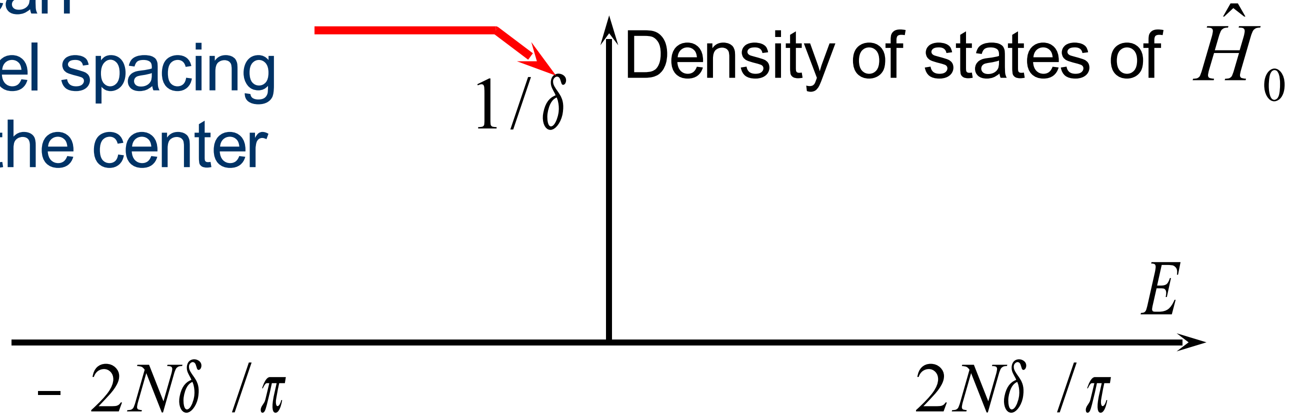
All orbitals are connected: resonance between remore orbitals on arbitrary remote sites is possible

Random matrix theory

$$\hat{H}(t) = \hat{H}_0 + \hat{V}\phi(t)$$

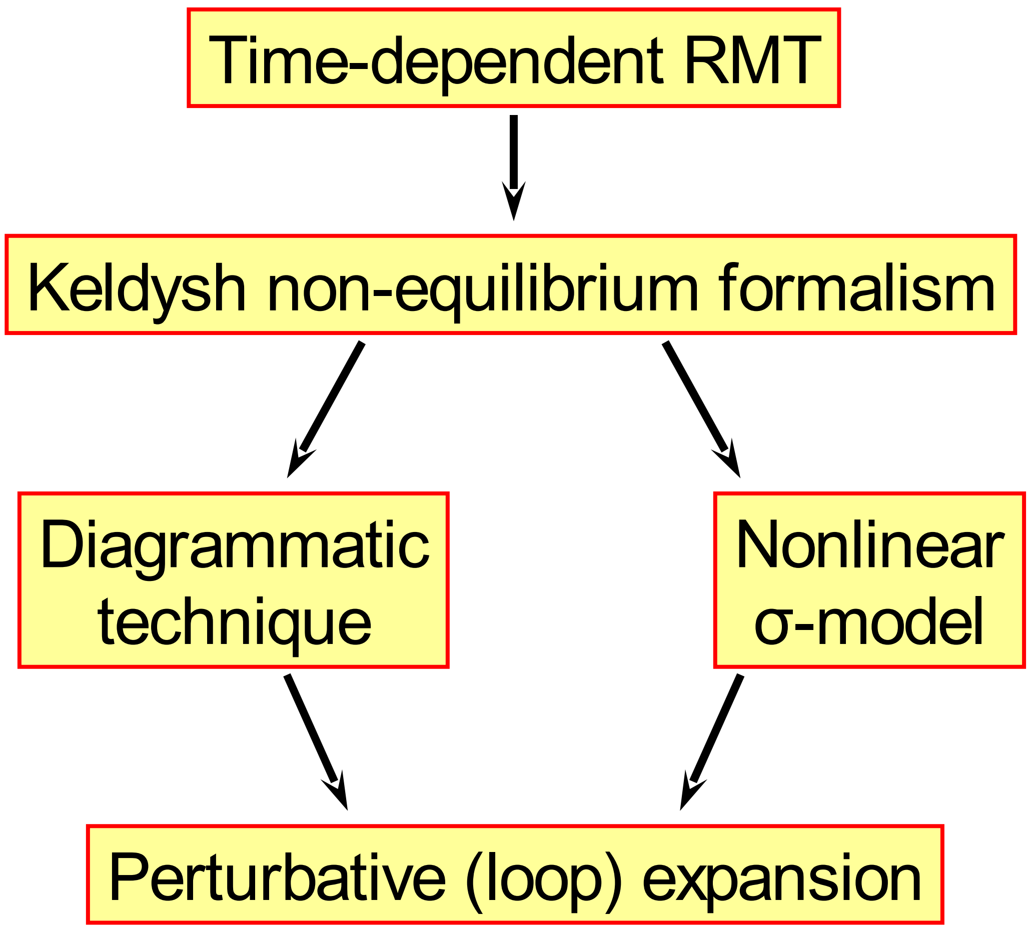
real symmetric
$N \times N$ Gaussian
random matrices
with statistically independent elements

mean
level spacing
at the center



In the end let $N \to \infty$

Technicalities



Zero order (diffusion)

$\Gamma \equiv \left\langle V_{ll'}^2 \right\rangle / \delta$ – one photon absorption rate (measure of perturbation strength)

Long-time, period-averaged dynamics:

$$\left[\frac{\partial}{\partial t} - D \frac{\partial^2}{\partial E^2} \right] f(E,t) = 0$$

time-dependent electron distribution (Wigner variables)

$D = \Gamma \, \overline{(d\phi \, / \, dt)^2}$ – energy diffusion coefficient

$$W_0 \equiv \frac{\partial}{\partial t} \int E \, f(E,t) \, dE = \frac{D}{\delta}$$

– energy absorption rate

One-loop correction

$$W(t) = \frac{D}{\delta} + \frac{\Gamma}{\pi}\int_0^t \dot{\phi}(t)\,\dot{\phi}(t-\tau)\,C_{t-\tau/2}(\tau,-\tau)\,d\tau$$

large zero-order

small (?) correction

Cooperon keeps track of the quantum interference:

$$C_t(\tau_1,\tau_2) \equiv \theta(\tau_1-\tau_2)\exp\left[-\int_{\tau_2}^{\tau_1}\frac{\Gamma}{2}[\phi(t+\tau/2)-\phi(t-\tau/2)]^2\,d\tau\right]$$

dephasing rate

Periodic perturbation

$$\phi(t) = \sum_{n=1}^{\infty} A_n \cos(n\omega t - \varphi_n) \qquad W_0 = \frac{\Gamma \omega^2}{2\delta} \sum_n n^2 A_n^2$$

$$C_t(\tau_1, \tau_2) \approx \exp\left[-\Gamma(\tau_1 - \tau_2) \sum_{n=1}^{\infty} A_n^2 \sin^2(n\omega t - \varphi_n) \right]$$

If $\varphi_n = n\varphi$ the exponent can vanish at $t_k = \dfrac{\varphi + k\pi}{\omega}$

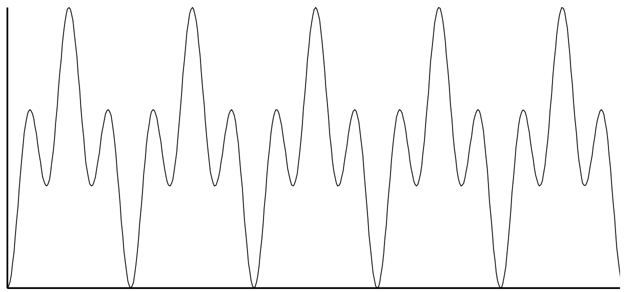
No-dephasing points give a large negative contribution to the integral:

$$W(t) - W_0 \sim -\omega^2 \sqrt{\Gamma t}$$

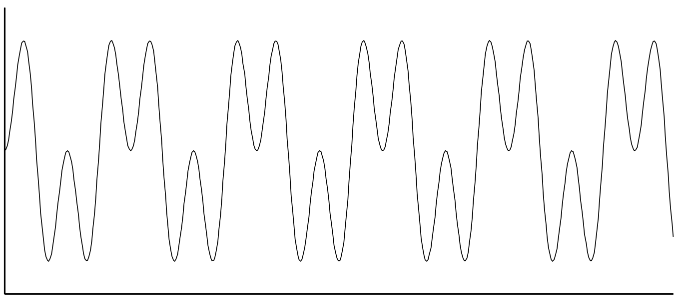
Time-reversal symmetry

$$\varphi_n = n\varphi \quad \Leftrightarrow \quad \phi(t - t_0) = \phi(-t - t_0)$$

Average dephasing rate versus time:



T-symmetry: yes



T-symmetry: no

Monochromatic perturbation: *T*-symmetry always –
a very special case

Two loops

There is a contribution from diffusons:

$$D_\tau(t_1,t_2) \equiv \theta(t_1 - t_2)\exp\left[-\int_{t_2}^{t_1} \Gamma\,[\phi(t+\tau/2) - \phi(t-\tau/2)]^2\,dt \right]$$

For a periodic perturbation:

$$D_\tau(t_1,t_2) \approx \exp\left[-2\Gamma(t_1 - t_2)\sum_{n=1}^{\infty} A_n^2 \sin^2 n\omega\tau \right]$$

No-dephasing points are always present, regardless of the time-reversal symmetry…

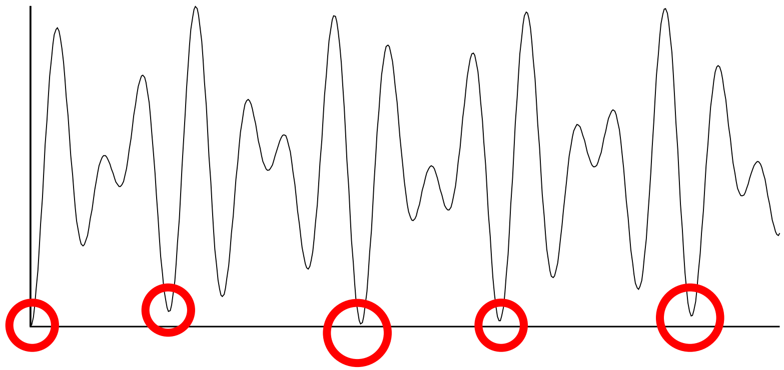
$$W(t) - W_0 = -\frac{\omega^2\delta}{24\pi^2}t$$

Incommensurate periods

$$f(t) = \sum_{n=1}^{d} A_n \cos(\omega_n t - \varphi_n)$$

$$\gamma_c = \sum_n \sin^2(\omega_n t + \varphi_n) A_n^2$$

dephasing rate:



Phase relationships do not matter that much

Almost-no-dephasing points contribute:

$$W(t) - W_0 \sim -\omega^2 \int_{1/\Gamma}^{t} \frac{\Gamma\, dt_1}{\sqrt{(\Gamma t_1)^d}}$$

d-dimensional weak Anderson localization

Conclusions…

1. A quantum-mechanical system under a time-dependent perturbation may be subject to dynamic localization in energy space.

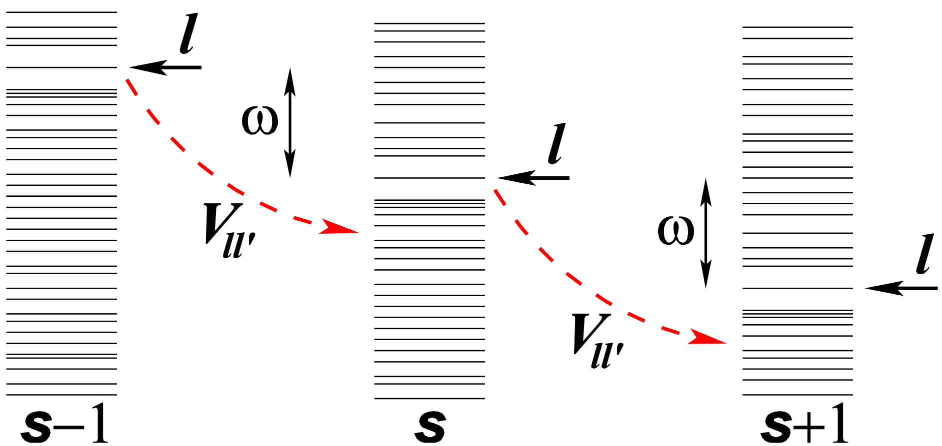
3. It depends both on the model for the unperturbed system and the perturbation.

5. We have studied one-loop correction to the usual Fermi-Golden-Rule dissipation rate for a chaotic system described by RMT

…conclusions

1. For a perturbation with *d* incommensurate frequencies the correction can grow arbitrarily with time if $d=1,2$ (analogously to spatial localization in *d*-dimensional disorder)
2. For commensurate frequencies phase relationships matter:
3. Time-reversal symmetry: the “dimensionality” is effectively lowered
4. No time-reversal: the correction is small

A stationary analogy



following directly from Schrödinger equation for $\hat{H}_0 + \hat{V}\cos\omega t$

- Take the original levels E_l of $\hat{H}_0$
- Replicate them into a lattice with a shift

$$E_{l,s} = E_l - s\hbar\omega$$

- Couple neighboring sites with $\hat{V}$

Why RMT is not KQR

- Quantum rotor: $\psi_l = e^{il\theta}$, $V(\theta) = \cos\theta$,

  $V_{ll'} \propto \delta_{l',l\pm 1}$ – out of resonance

  $V(t) \propto \delta(t - nT)$ – all Fourier harmonics $V^{(s-s')}$

- Particle in a box: $\psi_l = \sin l\theta$, $V(\theta) \propto \cos\theta$,

  $V_{ll'} \propto 1/|l - l'|$ – long-range

- Random matrix: $V_{ll'} \propto const$ but

  we want few Fourier harmonics $V^{(s-s')}$

Spatial localization

Quantum correction to the diffusion coefficient of electrons in disorder

$$D = D_0 - \frac{1}{\nu}\int_{1/L}^{1/l} \frac{d^d \vec{k}}{k^2}$$

mean free path

density of states

sample size

$d = 1: \quad L_{loc} \sim \nu D_0 \sim l$

$d = 2: \quad L_{loc} \sim l \exp(\nu D_0)$ (?)

$d \geq 3:$ no localization in weak disorder