Magnetoconductance oscillations in networks of diffusive wires & Decoherence due to electron-electron interaction

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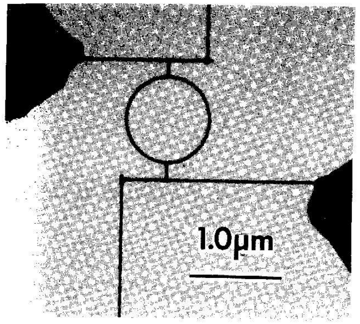
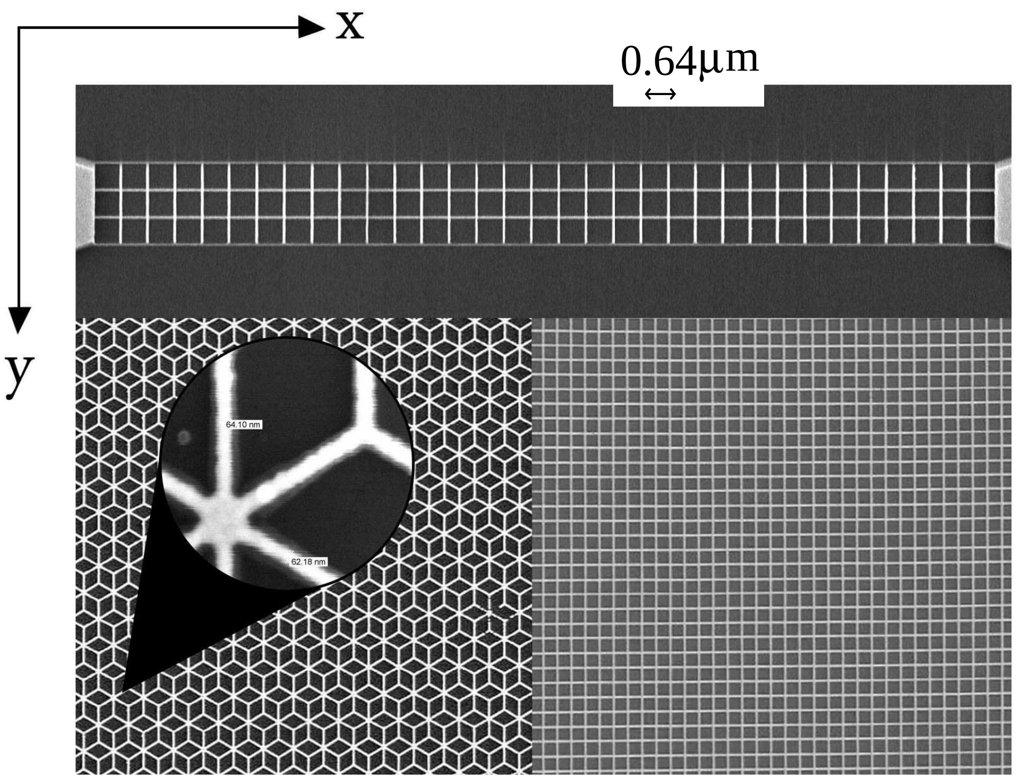
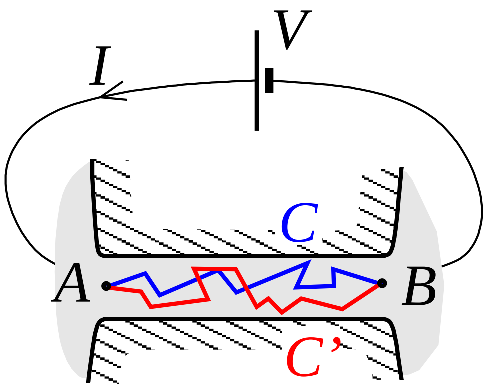
- Washburn & Webb (1985) : gold rings



- Bäuerle & Saminadayar (2005) : silver networks



Conductance of a weakly disordered metal



$$G = \frac{I}{V} \sim \overbrace{\left| \sum_{\mathcal{C}} \mathcal{A}_{\mathcal{C}}(A \to B) \right|^2}^{\text{Proba}(A \to B)}$$

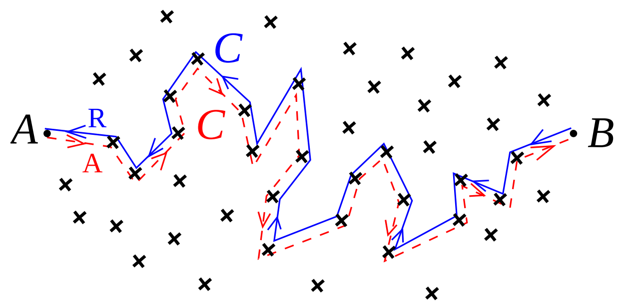
$$= \sum_{\mathcal{C}} |\mathcal{A}_{\mathcal{C}}|^2 + \sum_{\mathcal{C} \neq \mathcal{C}'} \mathcal{A}_{\mathcal{C}} \mathcal{A}^*_{\mathcal{C}'}$$

amplitude $\mathcal{A}_{\mathcal{C}} \sim e^{ik_F l(\mathcal{C})}$ has a large random phase

Average conductance : $\langle G \rangle = \underbrace{G_{\text{Drude}}}_{\text{classical}} + \underbrace{\langle \Delta G \rangle}_{\text{quantum correction}}$

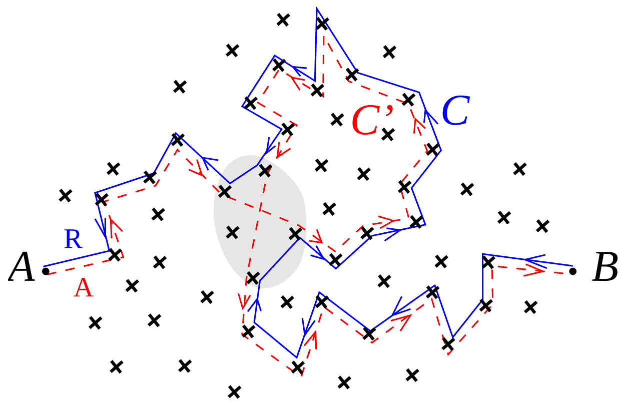
Classical transport : $\boxed{\mathcal{C} = \mathcal{C}'}$

$$G_{\mathrm{Drude}} = \sigma_0 \frac{\text{section}}{\text{length}} \sim \langle \sum_{\mathcal{C}} |\mathcal{A}_{\mathcal{C}}|^2 \rangle$$



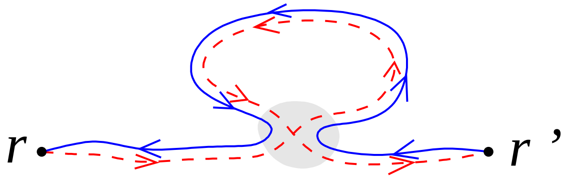
Quantum interferences : $\boxed{\mathcal{C} \neq \mathcal{C}'}$

Weak localization correction $\langle \Delta G \rangle$



Weak localization correction

- Quantum crossing $\Rightarrow$ a small correction



- Increase of backscattering : WL for the wire is $\langle \Delta G \rangle < 0$
- COHERENT contribution : loops smaller than L_φ contribute

Experimental probe of phase coherence

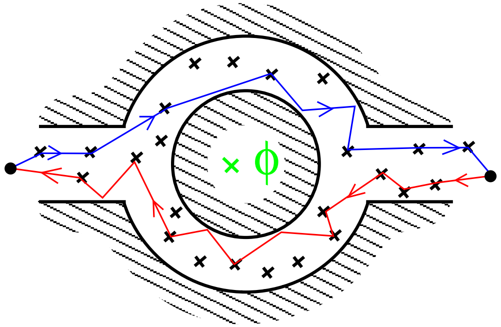
- Magnetic field sensitivity : $\langle \Delta G(\mathcal{B}) \rangle$

Contents

- AAS oscillations in isolated ring
- Effect of arms in a connected ring
- Chain of rings
- Decoherence due to electron-electron interaction
  ▷ application to single ring and chain
- Summary

AB / AAS oscillations

- Aharonov-Bohm (AB) oscillations

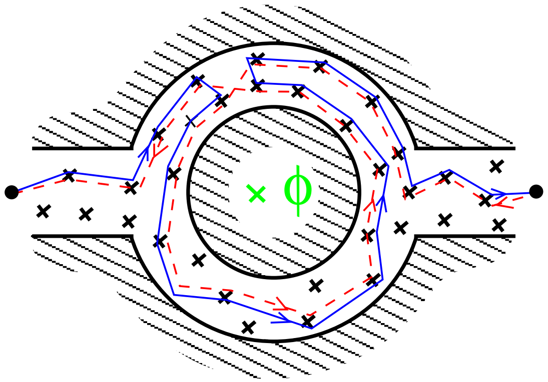


$$\Rightarrow \mathcal{A}_{\mathcal{C}}\mathcal{A}^*_{\mathcal{C}'} \propto \mathrm{e}^{\mathrm{i}e\phi/\hbar}$$

$G(\phi)$: h/e AB oscillations

$\Downarrow$ disorder averaging

- Al'tshuler-Aronov-Spivak (AAS) oscillations



$$\Rightarrow \mathcal{A}_{\mathcal{C}}\mathcal{A}^*_{\mathcal{C}'} \propto \mathrm{e}^{2\mathrm{i}e\phi/\hbar}$$

$\langle \Delta G(\phi)\rangle$: $h/2e$ AAS oscillations

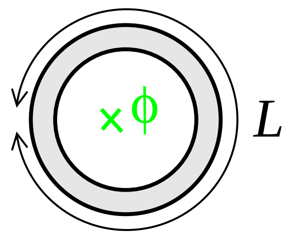
AAS oscillations (2)

$$\langle \Delta\sigma \rangle = -\frac{e^2}{\pi}\int_0^\infty \mathrm{d}t\, \mathcal{P}(x,x;t)\, \mathrm{e}^{-t/\tau_\varphi}$$

$$\text{where } \left(\partial_t - [\nabla_x - 2\mathrm{i}eA]^2\right)\mathcal{P}(x,x';t) = \delta(t)\delta(x-x')$$

Harmonics of MC in a ring :

$$\langle \Delta\sigma(\theta) \rangle = \sum_n \langle \Delta\sigma_n \rangle\, \mathrm{e}^{\mathrm{i}n\theta} \text{ with } \theta = 4\pi\frac{\phi}{\phi_0}$$



$$\langle \Delta\sigma_n \rangle = -\frac{e^2}{\pi}\int_0^\infty \mathrm{d}t \underbrace{\mathcal{P}_n(x,x;t)}_{\text{Proba to wind } n \text{ times}} \mathrm{e}^{-t/\tau_\varphi}$$

$$= -\frac{e^2}{\pi}\int_0^\infty \mathrm{d}t\, \frac{1}{\sqrt{4\pi t}}\, \mathrm{e}^{-\frac{(nL)^2}{4t}}\, \mathrm{e}^{-t/\tau_\varphi} = \boxed{-\frac{e^2}{h} L_\varphi \exp -|n|L/L_\varphi}$$

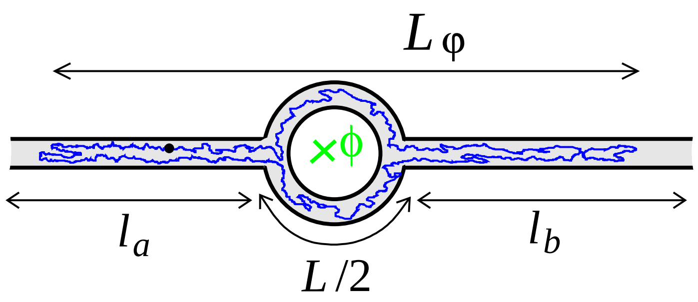
Nonlocality – Effect of connecting wires

C.T. & G. Montambaux, J.Phys.A**38** (2005)

The arms strongly manifest for $L_\varphi \gtrsim L$

For $l_a,\, l_b \gg L_\varphi \gg L$:

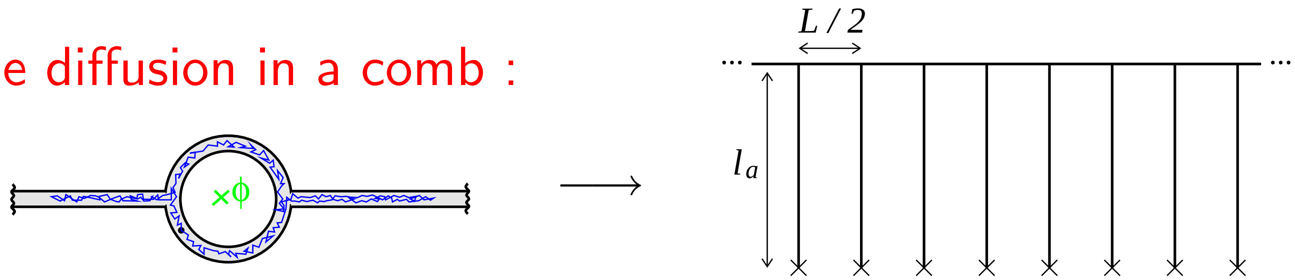
$$\langle \Delta g_n \rangle \simeq -\frac{1}{\sqrt{2}} \frac{L^{1/2} L_\varphi^{3/2}}{(l_a + l_b)^2} \, \mathrm{e}^{-|n|\sqrt{2L/L_\varphi}}$$



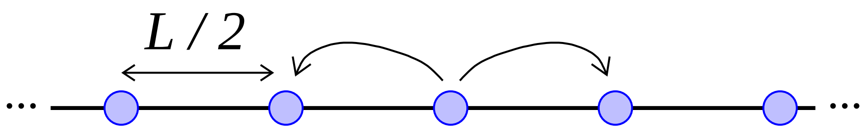
Winding is slow : $n_t \propto t^{1/4}$

(isolated ring : $n_t \propto t^{1/2}$)

Mapping to the diffusion in a comb :



trapping by the arms :



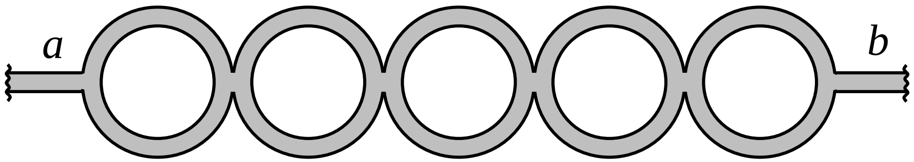
Trapping time distribution :

first return probability in 1d : $Q(t) \sim 1/t^{3/2}$ $\longrightarrow n_t \sim t^{1/4}$

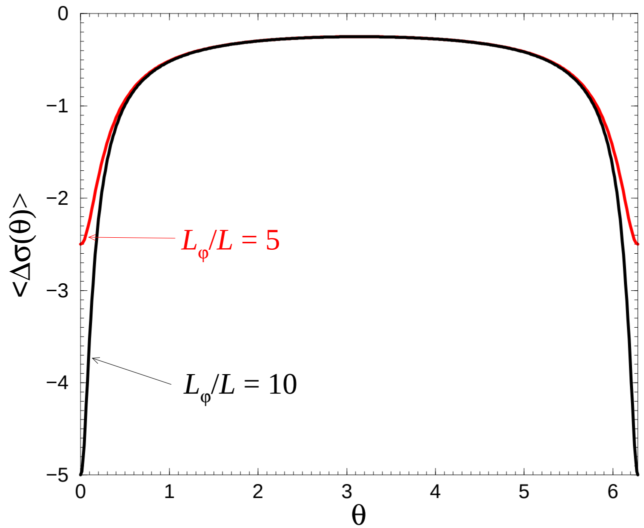
$$\boxed{\mathcal{P}_n(x,x;t) \simeq \frac{\sqrt{L/2}}{2\,t^{3/4}}\;\psi\!\left(\frac{n\sqrt{2L}}{t^{1/4}}\right)}$$

$$\psi(0) = \frac{\Gamma(3/4)}{\pi\sqrt{2}} \quad \text{and} \quad \psi(\xi) \underset{\xi\gg 1}{\simeq} \frac{4}{\sqrt{6\pi}}(\xi/4)^{1/3}\mathrm{e}^{-3(\xi/4)^{4/3}}$$

Chain of rings (for $L_\varphi/L \gg 1$)



$$\langle\Delta\tilde{\sigma}_{\rm osc}(\theta)\rangle = -\frac{L_\varphi}{2}\frac{\sinh(L/2L_\varphi)}{\sqrt{\cosh^2(L/2L_\varphi)-\cos^2(\theta/2)}}$$



$$\langle\Delta\tilde{\sigma}_n\rangle \simeq -L\int_0 \frac{\mathrm{d}\theta}{2\pi}\frac{\mathrm{e}^{-\mathrm{i}n\theta}}{\sqrt{(\frac{L}{L_\varphi})^2+\theta^2}} \simeq -\frac{L}{2\pi}\int_{L/L_\varphi}\frac{\mathrm{d}\theta}{\theta}$$

$$\boxed{\langle\Delta\tilde{\sigma}_n\rangle \simeq -\frac{L}{2\pi}\ln(L_\varphi/L) + \mathrm{cste}}$$

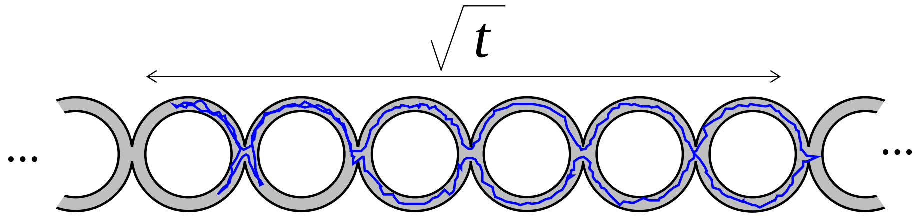
Chain of rings (2)

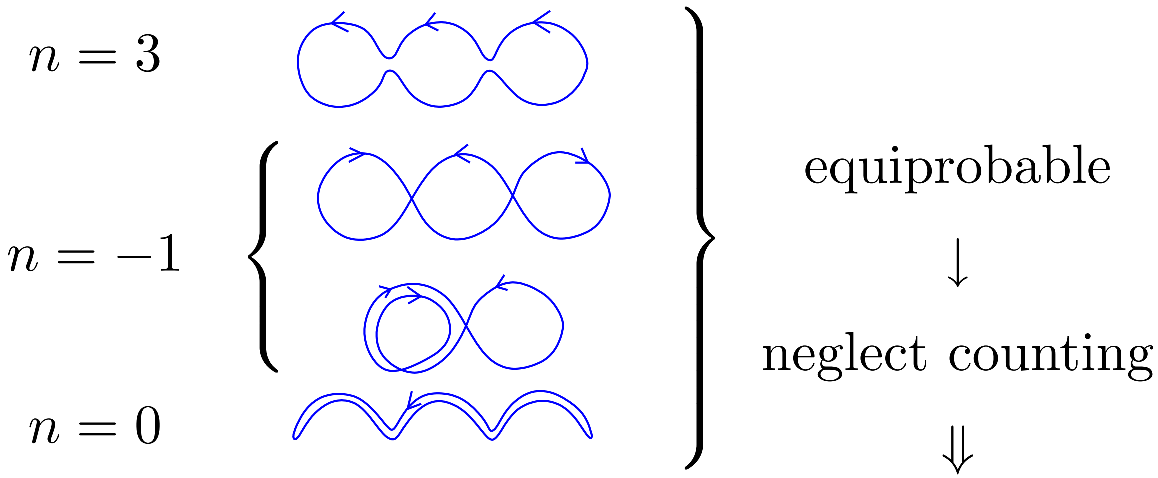
$$\begin{aligned}\langle \Delta g_n \rangle &\simeq -\frac{1}{N_r}\,\frac{2}{\pi}\left[\ln(2L_\varphi/|n|L) + b_{|n|}\right] \\ &\simeq -\frac{1}{N_r}\,\frac{\mathrm{e}^{-|n|\,L/L_\varphi}}{\sqrt{\pi|n|\,L/2L_\varphi}} \qquad \text{for } |n| \gg L_\varphi/L\end{aligned}$$

$\big\downarrow$ Inverse Laplace

$$\boxed{\mathcal{P}_n(x,x;t) \simeq \frac{L}{8\pi t}\mathrm{e}^{-(nL)^2/4t}} \quad \text{for } t \gg L^2$$

Explanation for $\mathcal{P}_n(x,x;t) \simeq \frac{L}{8\pi t} \mathrm{e}^{-(nL)^2/4t}$





$$\sum_n \mathcal{P}_n(x,x;t) = \frac{1}{2\sqrt{4\pi t}} \quad \Rightarrow \quad \mathcal{P}_n(x,x;t) \sim \frac{L}{t} \text{ for } |n| \lesssim \frac{\sqrt{t}}{L}$$

Decoherence due to electron-electron interaction in networks

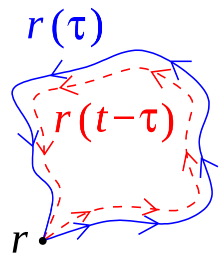
Exponential relaxation of phase coherence

$$\langle \Delta\sigma \rangle \sim - \sum_{\text{loops } \mathcal{C}_t} e^{-t/\tau_\varphi}$$

Decoherence due to e-e interaction : AAK's model

Al'tshuler, Aronov & Khmel'nitzkiĭ, J.Phys.C**15** (1982).

$$\langle \Delta\sigma \rangle \sim - \sum_{\text{loops } \mathcal{C}_t} \langle e^{i\Phi[\mathcal{C}_t]} \rangle_V$$



$$\Phi[r(\tau)] = \int_0^t d\tau \, [V(r(\tau),\tau) - V(r(\tau), t-\tau)]$$

$V(r,t)$ is the fluctuating electrostatic potential

Decoherence due to electron-electron interaction (2)

$$\langle \Delta\sigma_n \rangle = -\frac{e^2}{\pi}\int_0^\infty \mathrm{d}t\, \mathcal{P}_n(x,x;t) \underbrace{\langle \mathrm{e}^{\mathrm{i}\Phi} \rangle_{V,\mathcal{C}_n}}_{\text{replaces } \mathrm{e}^{-t/\tau_\varphi}}$$

Relaxation of phase coherence :

$$\langle \mathrm{e}^{\mathrm{i}\Phi} \rangle_{V,\mathcal{C}_n} = \left\langle \mathrm{e}^{-\frac{2}{L_N^3}\int_0^t \mathrm{d}\tau\, W(x(\tau),x(t-\tau))} \right\rangle_{\mathcal{C}_n}$$

$$W(x,x') = \frac{P_d(x,x)+P_d(x',x')}{2} - P_d(x,x') \text{ with } -\Delta P_d = \delta$$

Nyquist length : $L_N = \left(\frac{\sigma_0 S D}{e^2 T}\right)^{1/3} = \left(\frac{\alpha_d}{\pi} N_c \ell_e L_T^2\right)^{1/3}$ with $L_T = \sqrt{\frac{D}{T}}$

$\Rightarrow$ decoherence depends on
- network
- trajectories

Decoherence in the infinite wire

Al'tshuler, Aronov & Khmel'nitzkiĭ, J.Phys.C**15** (1982).

$$\langle \Delta\sigma_n \rangle = \frac{e^2}{h} L_N \frac{\mathrm{Ai}(L_N^2/L_\varphi^2)}{\mathrm{Ai}'(L_N^2/L_\varphi^2)} = -\frac{e^2}{\pi} \int_0^\infty \frac{\mathrm{d}t}{\sqrt{4\pi t}} \langle \mathrm{e}^{\mathrm{i}\Phi} \rangle_{V,\mathcal{C}}$$

Relaxation of phase coherence

Montambaux & Akkermans, PRL**95** (2005).

$$\langle \mathrm{e}^{\mathrm{i}\Phi} \rangle_{V,\mathcal{C}} \simeq 1 - \frac{\sqrt{\pi}}{4} \left(\frac{t}{\tau_N}\right)^{3/2} \quad \text{for } t \ll \tau_N$$

$$\simeq \frac{1}{|u_1|} \sqrt{\frac{\pi t}{\tau_N}}\, \mathrm{e}^{-|u_1| t/\tau_N} \quad \text{for } t \gg \tau_N$$

with $u_1 \simeq -1.019$

Decoherence in the isolated ring (1)

$$\langle \Delta\sigma_n(L_\varphi, L_N)\rangle = \overbrace{\frac{e^2}{h}\, L_N\, \frac{\mathrm{Ai}(L_N^2/L_\varphi^2)}{\mathrm{Ai}'(L_N^2/L_\varphi^2)}}^{\mathrm{AAK}}\ \exp -|n|\ell_{\mathrm{eff}}$$

C.T. & G. Montambaux, PRB**72** (2005).

- Combination of L_φ and L_N : $\quad \frac{1}{\tau_\varphi} \longrightarrow \cancel{\frac{1}{\tau_\varphi} + \frac{1}{\tau_N}}$

$$\ell_{\mathrm{eff}} = \left(\frac{L}{L_N}\right)^{3/2} \times\ \eta(L_c^2/L_\varphi^2) \qquad \text{with} \qquad \boxed{L_c = \frac{L_N^{3/2}}{L^{1/2}}}$$

- $L_\varphi = \infty$: $\langle\Delta\sigma_n\rangle \propto \mathrm{e}^{-\frac{\pi}{8}|n|(\frac{L}{L_N})^{3/2}} \sim \boxed{\mathrm{e}^{-nL^{3/2}T^{1/2}}}$

Ludwig & Mirlin, PRB (2004)

Decoherence in the isolated ring (2)

Exponential relaxation of phase coherence

$$\langle \Delta\sigma_n \rangle = -\frac{e^2}{h} L_\varphi \, \mathrm{e}^{-|n| \frac{L}{L_\varphi}}$$

Decoherence due to e-e interaction

$$\langle \Delta\sigma_n \rangle \sim -\frac{e^2}{h} L_N \, \mathrm{e}^{-\frac{\pi}{8}|n|(\frac{L}{L_N})^{3/2}}$$

Relaxation of phase coherence in the ring (1)

- Diffusion of the phase : Johnson-Nyquist

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi = V$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\Phi^2\rangle_V = \int \mathrm{d}t\langle V(t)V(0)\rangle_V = 2e^2TR_t \sim e^2T\frac{r(t)}{\sigma_0 S} = \frac{r(t)}{\sqrt{D}\,\tau_N^{3/2}}$$

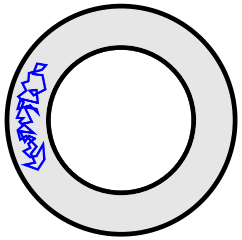
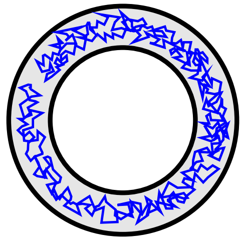
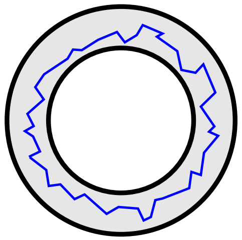
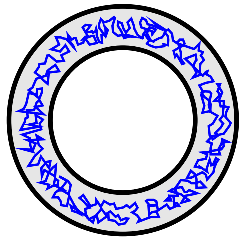
Small time $t \ll \tau_D$ $\Rightarrow r(t) \sim \sqrt{Dt}$ $\langle\Phi^2\rangle_V \approx (\frac{t}{\tau_N})^{3/2}$

Long time $t \gg \tau_D$ $\Rightarrow r(t) \sim L$ $\langle\Phi^2\rangle_V \approx \frac{\sqrt{\tau_D}}{\tau_N^{3/2}}\,t = \frac{t}{\tau_c}$

- For $\boxed{\tau_N \gg \tau_D}$: $\langle \mathrm{e}^{\mathrm{i}\Phi}\rangle_{V,\mathcal{C}_n} = \langle \mathrm{e}^{-\frac{1}{2}\langle\Phi^2\rangle_V}\rangle_{\mathcal{C}_n} \simeq \mathrm{e}^{-\frac{1}{2}\langle\Phi^2\rangle_{V,\mathcal{C}_n}}$

Relaxation of phase coherence in the ring (2)

$$\langle \mathrm{e}^{\mathrm{i}\Phi} \rangle_{V,\mathcal{C}_n} \simeq \mathrm{e}^{-\frac{1}{2}\langle \Phi^2 \rangle_{V,\mathcal{C}_n}} \ :$$

Harmonic	$t \ll \tau_D$		$t \gg \tau_D$	
$n = 0$		$\mathrm{e}^{-\frac{\sqrt{\pi}}{4}\left(\frac{t}{\tau_N}\right)^{3/2}}$ (AAK)		$\mathrm{e}^{-\frac{1}{6}\frac{t}{\tau_C}}$
$n \neq 0$		$\mathrm{e}^{-\frac{1}{6}\frac{t}{\tau_C}}$		$\mathrm{e}^{-\frac{1}{6}\frac{t}{\tau_C}}$

Relaxation of phase coherence in the ring (3)

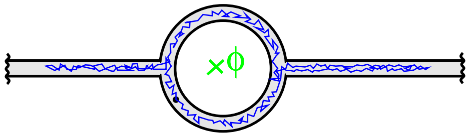
$$\langle \mathrm{e}^{\mathrm{i}\Phi} \rangle_{V,\mathcal{C}_n} \simeq \mathrm{e}^{-\frac{\pi^2}{64} t/\tau_c} \quad \text{for } n \neq 0$$

- Weak localization :

$$\langle \Delta\sigma_n \rangle \sim \int_0^\infty \mathrm{d}t \; \underbrace{\mathrm{e}^{-\frac{\pi^2}{64} t/\tau_c}}_{\text{phase relax.}} \; \underbrace{\frac{1}{\sqrt{t}} \mathrm{e}^{-(nL)^2/(4t)}}_{\text{diffusion}} \sim \mathrm{e}^{-\frac{\pi}{8} |n| (\frac{L}{L_N})^{3/2}}$$

$$\boxed{\langle \Delta\sigma_n \rangle \sim \mathrm{e}^{-n L^{3/2} T^{1/2}}} \qquad \text{for } L_N \ll L$$

Decoherence in the connected ring (for $L_N \gg L$)



If $x,\, x' \in \text{arms} \Rightarrow W(x,x') \simeq W_{\text{wire}}(x,x')$

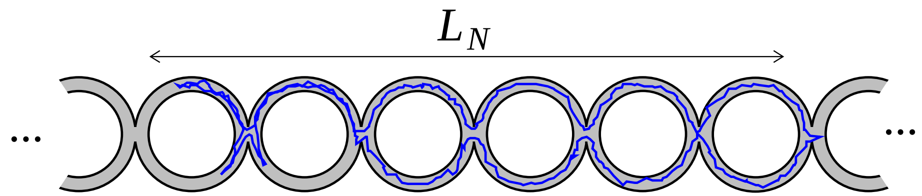
$$\langle \mathrm{e}^{\mathrm{i}\Phi} \rangle_{V,\mathcal{C}_n} \simeq \langle \mathrm{e}^{\mathrm{i}\Phi} \rangle_{V,\mathcal{C}} \Big|_{\text{wire}} \quad \text{and} \quad \mathcal{P}_n(x,x;t) \simeq \frac{\sqrt{L/2}}{2\,t^{3/4}}\, \psi\!\left(\frac{n\sqrt{2L}}{t^{1/4}}\right)$$

$$\Downarrow$$

$$\langle \Delta g_n \rangle \sim -\frac{L^{1/2} L_N^{3/2}}{(l_a+l_b)^2} \times \begin{cases} 1 & \text{for } n^2 \ll L_N/L \\ \left(\frac{n^2 L}{L_N}\right)^{7/12} \mathrm{e}^{-\kappa_2 |n| \sqrt{L/L_N}} & \text{for } n^2 \gg L_N/L \end{cases}$$

where $\kappa_2 = \sqrt{2}|u_1|^{1/4} \simeq 1.421$

Decoherence in the chain of N_r rings (for $L_N \gg L$)



If $x,\ x' \notin$ the same ring $\Rightarrow W(x,x') = \frac{1}{2} W_{\rm wire}(x,x')$

$$\langle e^{i\Phi} \rangle_{V,\mathcal{C}_n} \simeq \langle e^{i\Phi} \rangle_{V,\mathcal{C}} \Big|_{\rm wire}^{L_N^3 \to 2L_N^3} \quad \text{and} \quad \mathcal{P}_n(x,x;t) \simeq \frac{L}{8\pi t}\, e^{-(nL)^2/4t}$$

$$\Downarrow$$

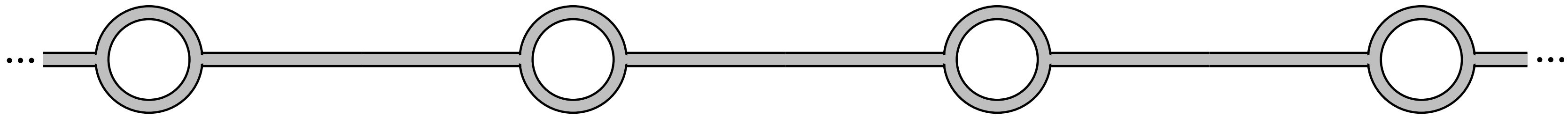
$$\langle \Delta g_n \rangle \simeq -\frac{1}{N_r} \times \begin{cases} \frac{2}{\pi}\ln(L_N/|n|L) & \text{for } |n| \ll L_N/L \\ \frac{1}{|u_1|^{3/2}}\, e^{-\kappa_3 |n| L/L_N} & \text{for } |n| \gg L_N/L \end{cases}$$

where $\kappa_3 = |u_1|^{1/2} 2^{-1/3} \simeq 0.801.$

Summary

	network	$-\langle \Delta g_n \rangle$	
$L_N \ll L$		$L_N\, e^{-n(L/L_N)^{3/2}}$	$\sim T^{-1/3}\, e^{-n\, L^{3/2} T^{1/2}}$
$L_N \gg L$	ring	$L_N^{11/12}\, e^{-n(L/L_N)^{1/2}}$	$\sim T^{-11/36}\, e^{-n\, L^{1/2} T^{1/6}}$
	chain	$e^{-n\, L/L_N}$	$\sim e^{-n\, L\, T^{1/3}}$

Experiment : compare $\langle \Delta g_n \rangle$ for



and