Dynamics and current fluctuations for AC forced charge shuttle

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F.P. Phys. Rev. B **69,** 245409 (2004),

F.P. and R. Fazio, Phys. Rev. Lett. **94**, 036806 (2005),

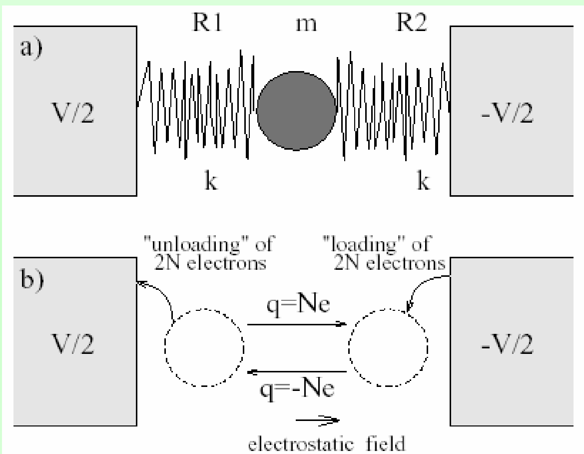
F.P. and R. Fazio, New Journal of Physics, special issue on NEMS (2006)

A new mechanism of electron transfer: charge Shuttle

VOLUME 80, NUMBER 20 PHYSICAL REVIEW LETTERS 18 MAY 1998

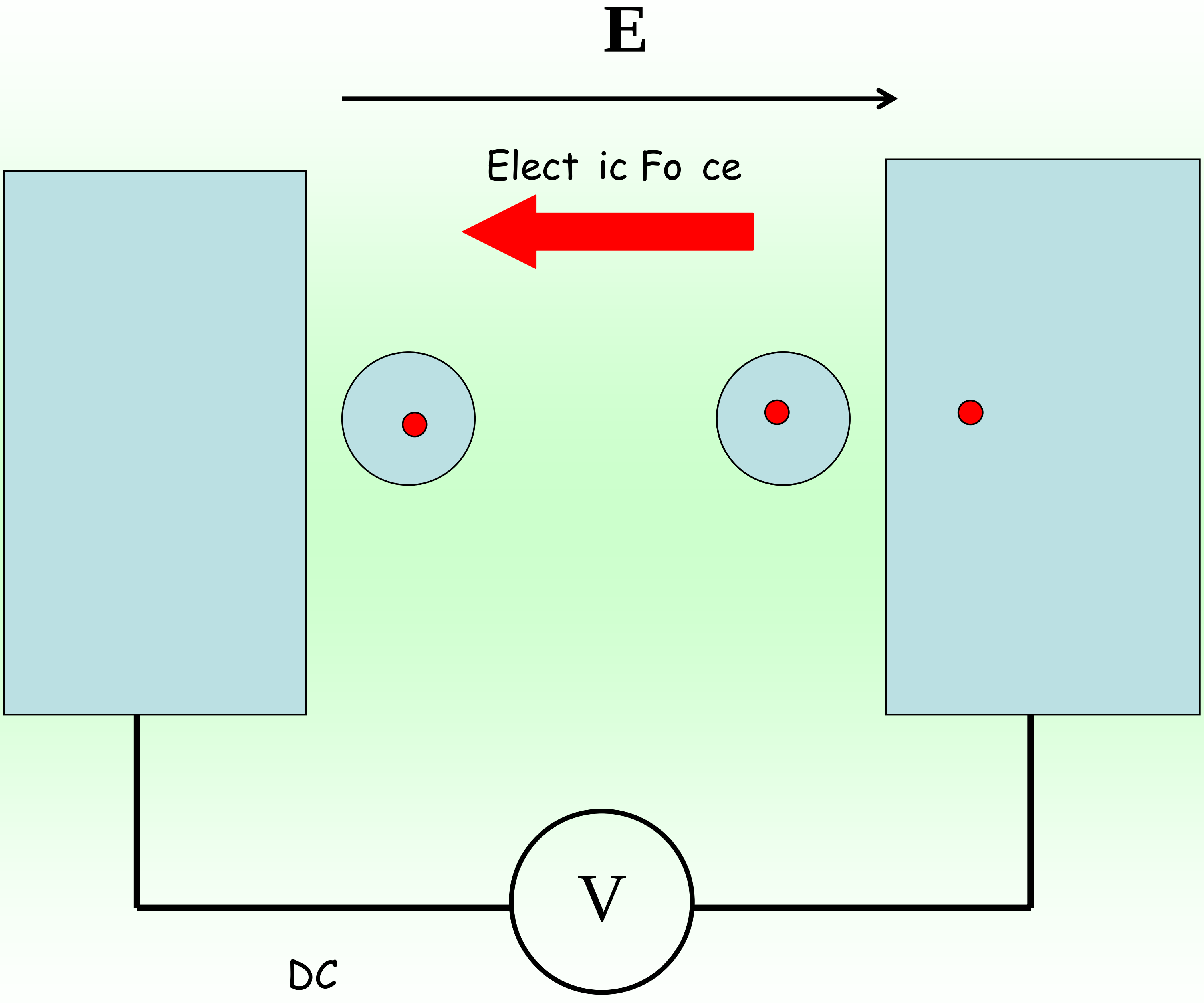
Shuttle Mechanism for Charge Transfer in Coulomb Blockade Nanostructures

L. Y. Gorelik,[1,2] A. Isacsson,[1] M. V. Voinova,[1,3] B. Kasemo,[1] R. I. Shekhter,[1] and M. Jonson[1]



Tunnelling esistance: $R(x)=R_o\, e^{-x/\lambda}$

See also Isacsson *et al* Physica (1998)


E
Elect ic Fo ce
V
DC

APPLIED PHYSICS LETTERS VOLUME 84, NUMBER 23 7 JUNE 2004

Silicon nanopillars for mechanical single-electron transport

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Robert H. Blick
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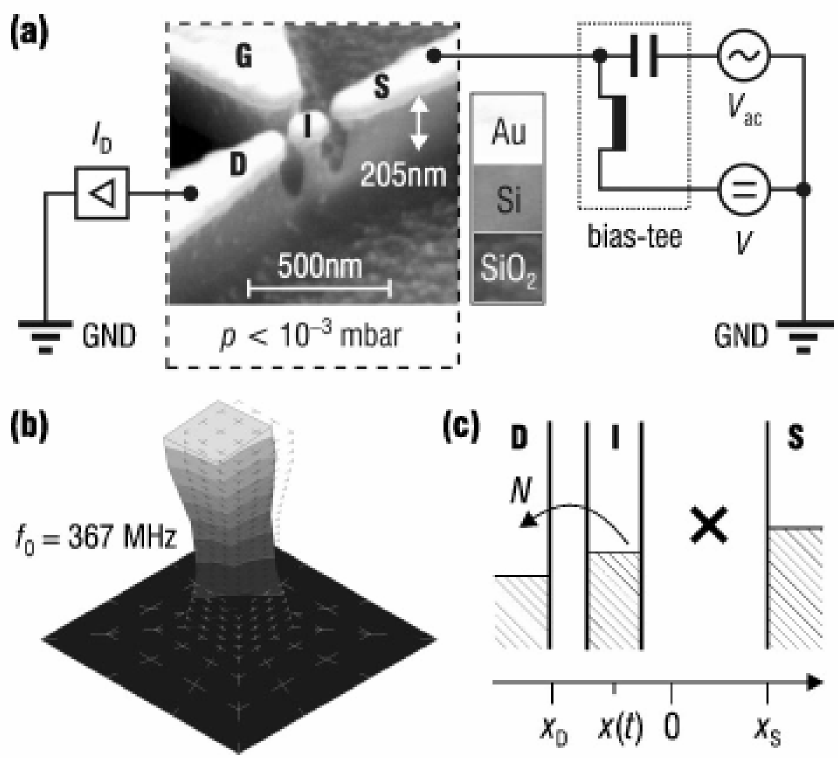


AC fo ced cha ge shuttle

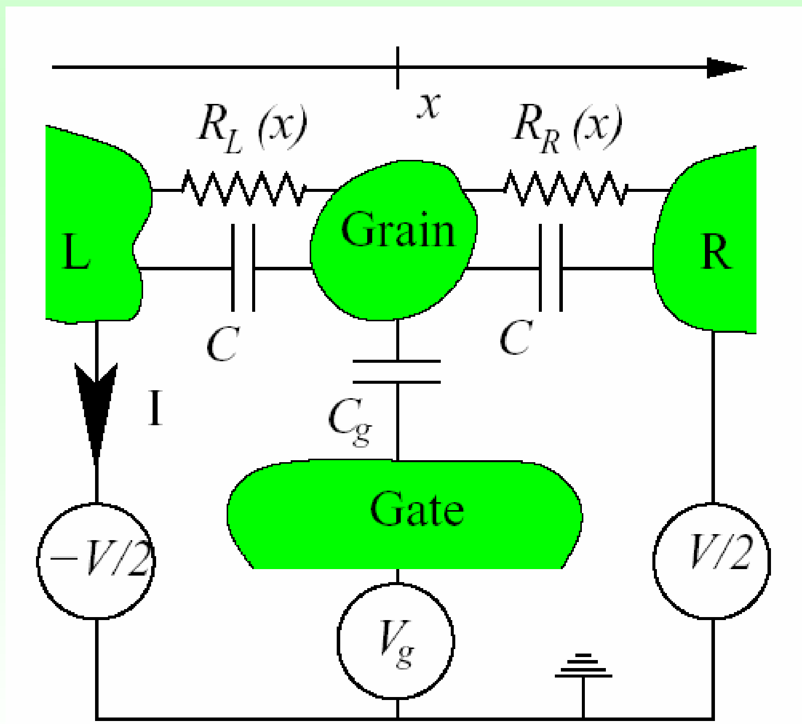
F.P. and R. Fa io, Phys. Rev. Lett. (2005)

Classical desc iption of the island:

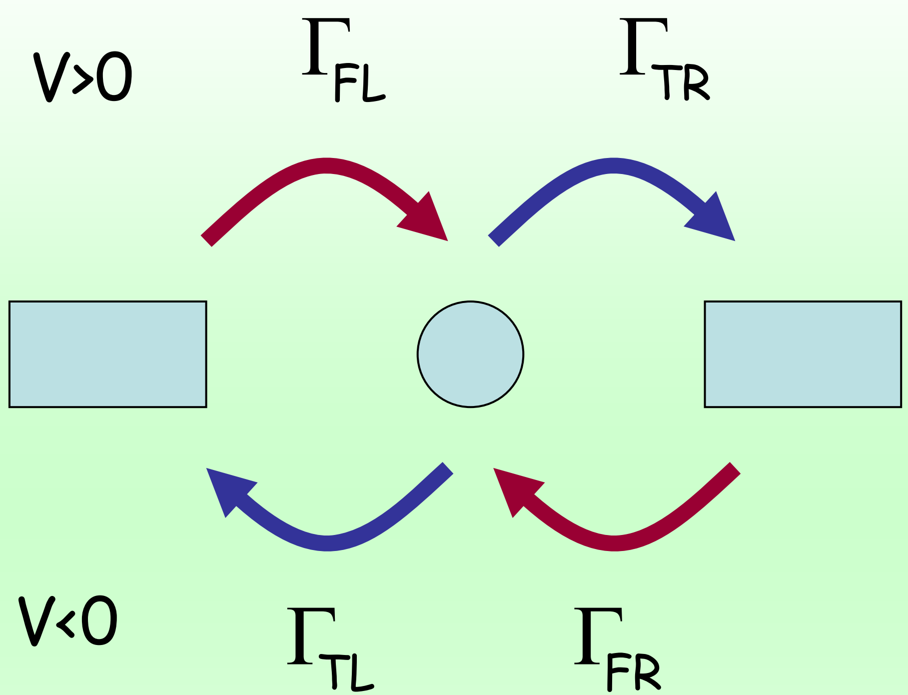
$$\ddot{x} = -\omega_o^2 x - \gamma\,\dot{x} + \frac{eV(t)}{Lm} n(t)$$

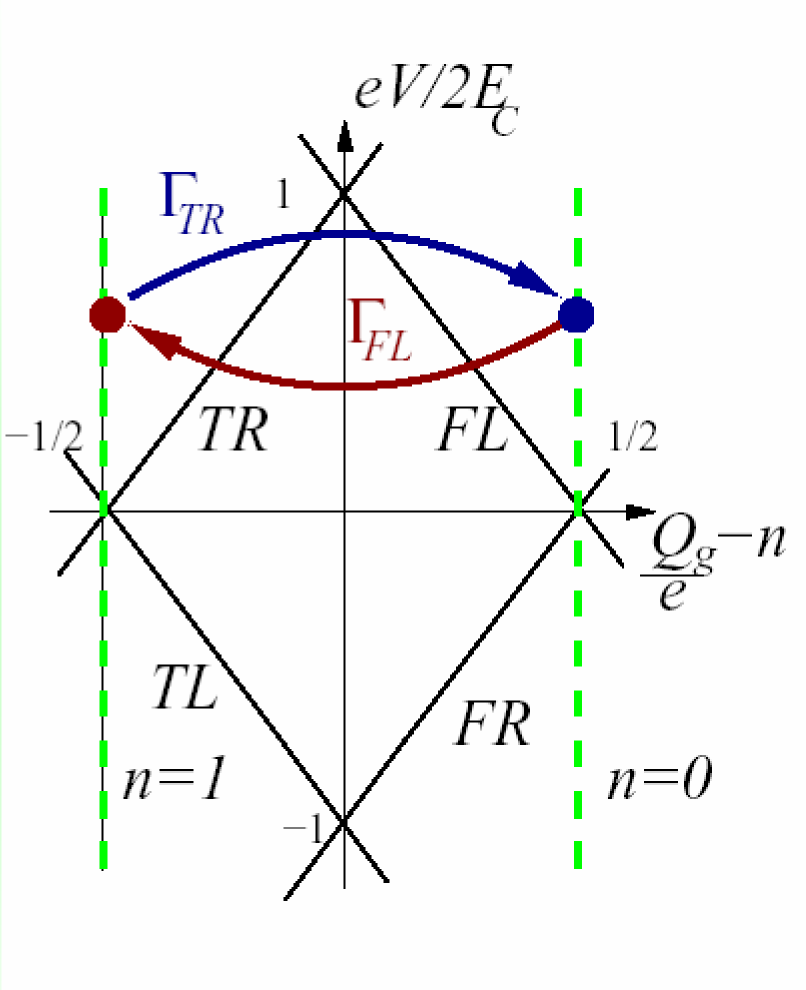
The fo ce depends on the stochastic va iable n(t)

Simple case, n=0,1. SET scheme



Coulomb blockade scheme





No memo y of the envi onment

Allows to use Γ(t)=Γ[x(t)]

$$\hbar\omega \ll k_B T \ll E_C \sim eV$$

The motion of the g ain influences the ates:

$$\Gamma_{FL}(t) = \frac{|eV(t)|}{4E_C}\,\Gamma_L[x(t)]\,\Theta[V(t)]$$

$$\Gamma_{L/R}(x) = [R_{L/R}(x)C]^{-1}$$

$$R_{R/L}(x) = R_{L/R}e^{\mp x/\lambda}$$

We find a Rectification effect fo the shuttle

Fo asymmet ic esistances ectification of the cu ent th ough the stochastic occupation of the g ain

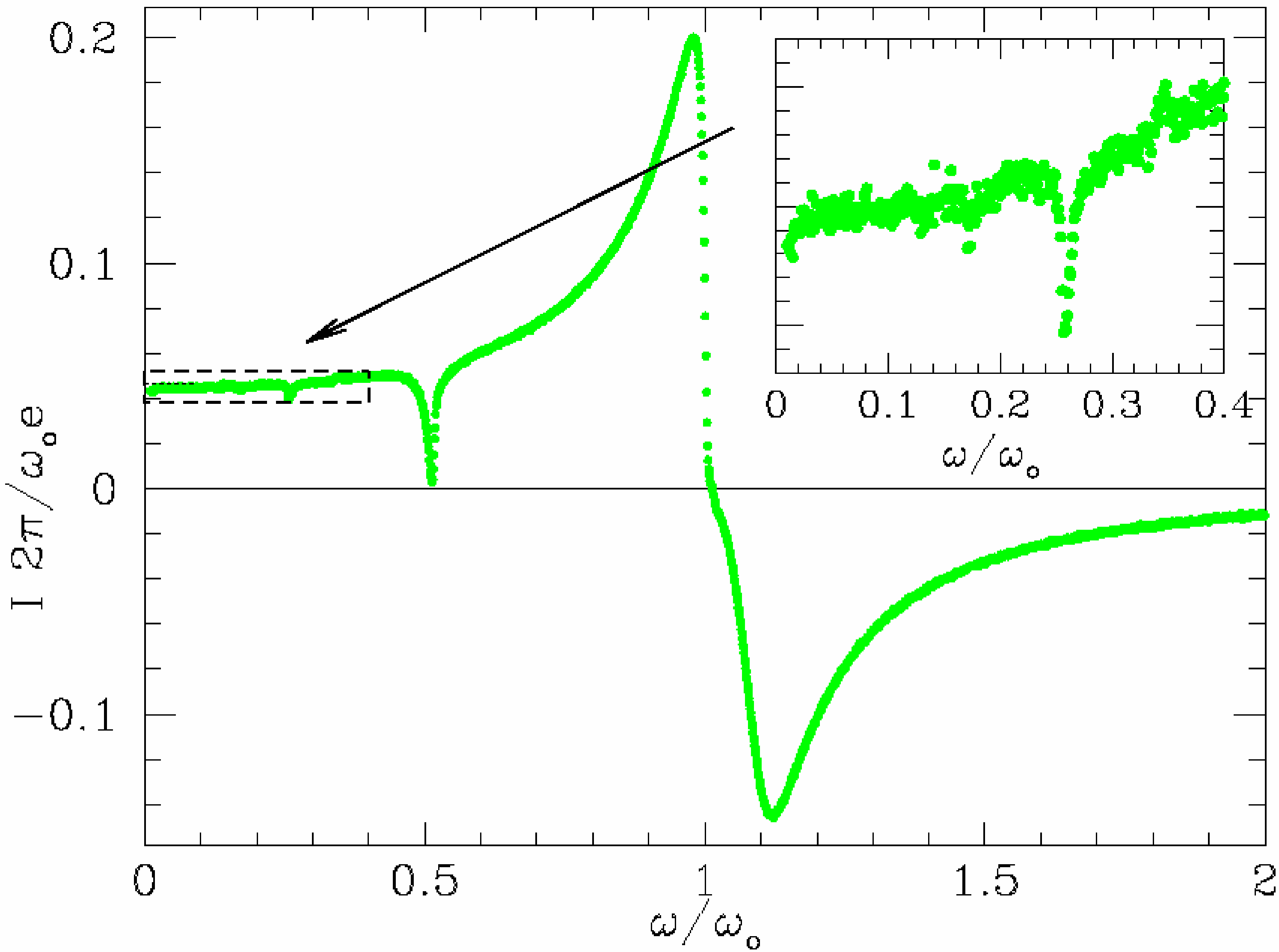
Evidence by Monteca lo simulations

We conside asymmet ic contacts (eg R_R= 10 R_L) and AC voltage bias

- analytical evolution of the g ain between two tunneling events
- tunneling events statistics dete mined by $\Gamma[x(t)]$
- each simulation 10^6 tunneling events

We let the voltage oscillate between $-E_C/e$ and $+E_C/e$ (only $n=\pm1$)

Rectified cu ent



Simplifying app oximation: mechanical fo ce $\gg$ elect ic fo ce

$$\epsilon = \frac{eE}{\lambda k} = \frac{\text{Electric Force}}{\text{Elastic Force}} \ll 1$$

The ha monic oscillation is only **slowly** modified by the elect ic field

stochastic fluctuations of the position a e then small

$$n(t) \rightarrow \langle n(t) \rangle = P_1(t)$$

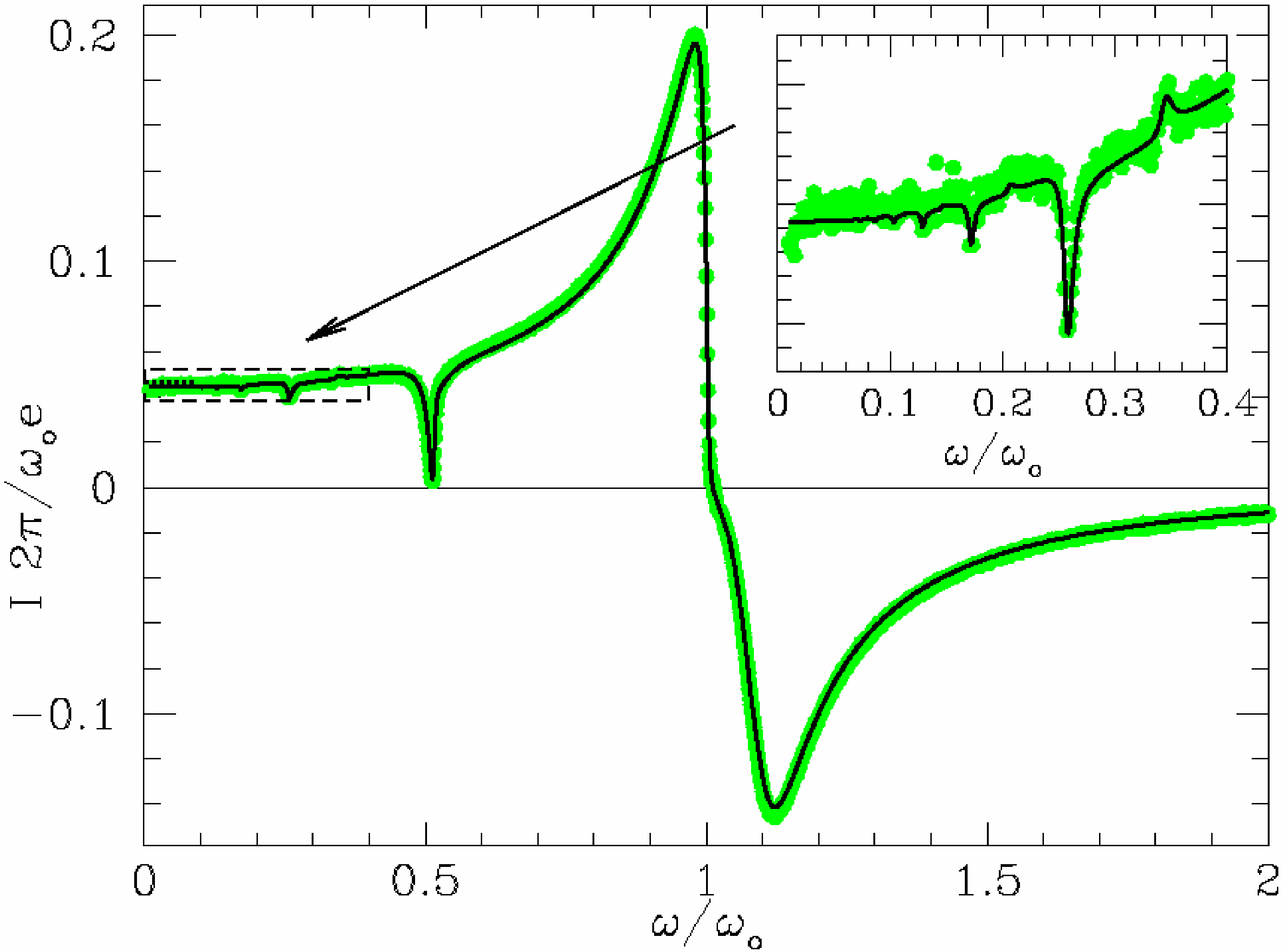
Maste equation fo P_1

$$\frac{dP_1}{dt} = -\Gamma_1[x(t)]\,P_1(t) + \Gamma_2[x(t)]$$

ave age cu ent

$$I(t)/e = [1 - P_1(t)]\Gamma_{FL}(t) - P_1(t)\Gamma_{TL}(t)$$

Black line: ave age app oximation



he e ε=0.5

Adiabatic limit $\omega \ll \omega_o$ and $\varepsilon\, \omega_o/\gamma$ < shuttling

The occupation of the g ain will be one that satisfies both stationa y maste equation and equilib ium of fo ces

$$P_1(x_e) = \Gamma_2(x_e)/\Gamma_1(x_e)$$

$$x_e = \frac{eV}{Lk} P_1(x_e) = \epsilon \lambda \frac{eV}{2E_C} P_1(x_e)$$

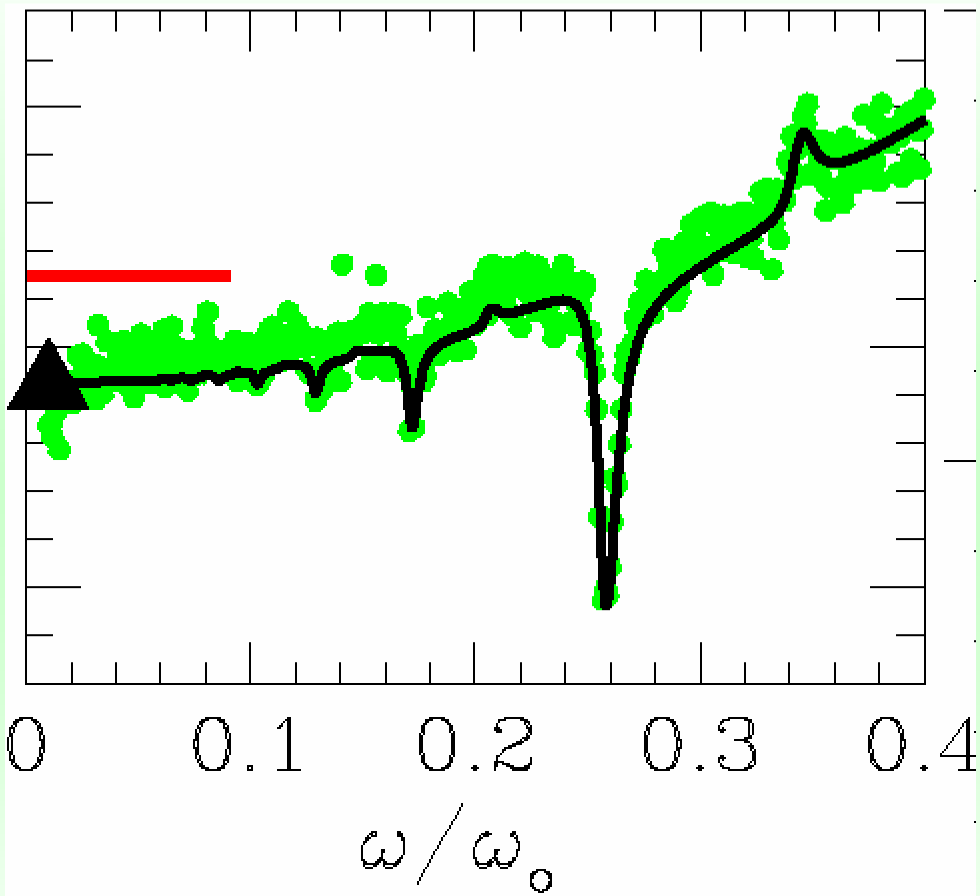
by pe tu bation theo y in ε

$$I_a(\omega \ll \omega_o) = \epsilon\, \frac{V_o e^2}{32E_C} \frac{\Gamma_L \Gamma_R (\Gamma_L - \Gamma_R)}{(\Gamma_R + \Gamma_L)^2}$$

I(V) $\neq$ I(-V) due to the motion of the g ain

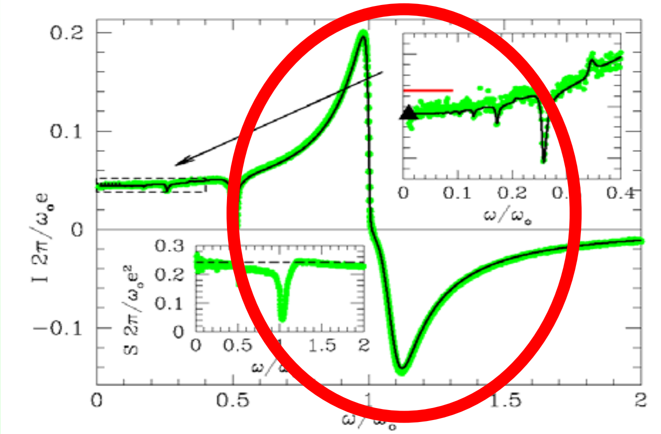
this can be a ve y simple and obust way of testing mechanical motion adiabatic esult

fo $\varepsilon \ll 1$



adiabatic
nume ical

Main esonance

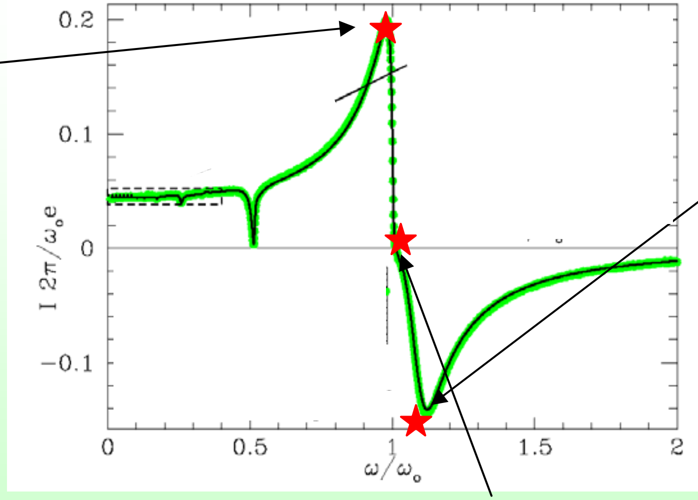


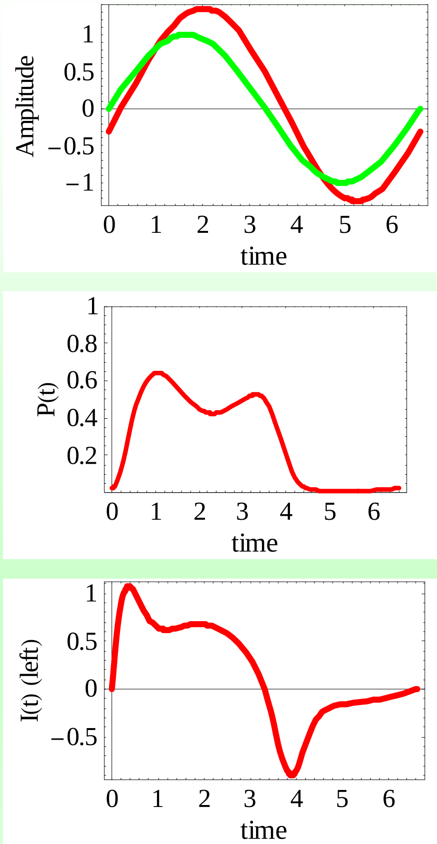
The f equency dependence can be unde stood in te ms of the standa d phase shift a ound a esonance

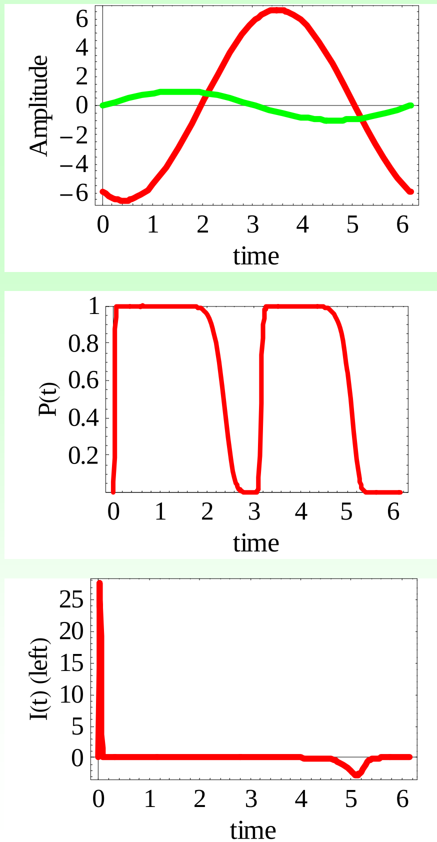
$$\phi(\omega) = \arctan[\omega\gamma/(\omega^2 - \omega_o^2)] \qquad x(t) = A\sin[\omega t - \phi(\omega)]$$

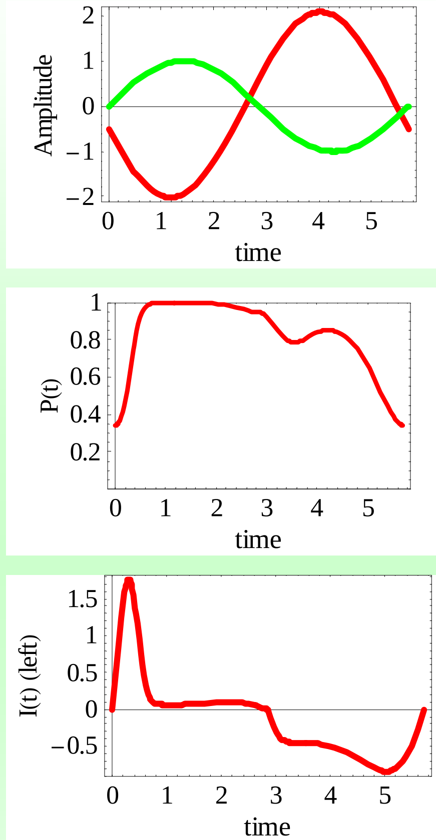
Solving the maste equation with pe iodic bounda y conditions and this fo m of x(t) one ep oduces the shape of the esonance

Phase shift cu ent dependence









Positive cu ent

ω/ω_o=.95

$\phi \approx 0$

~ vanishing I

$\phi \approx \pi/2$

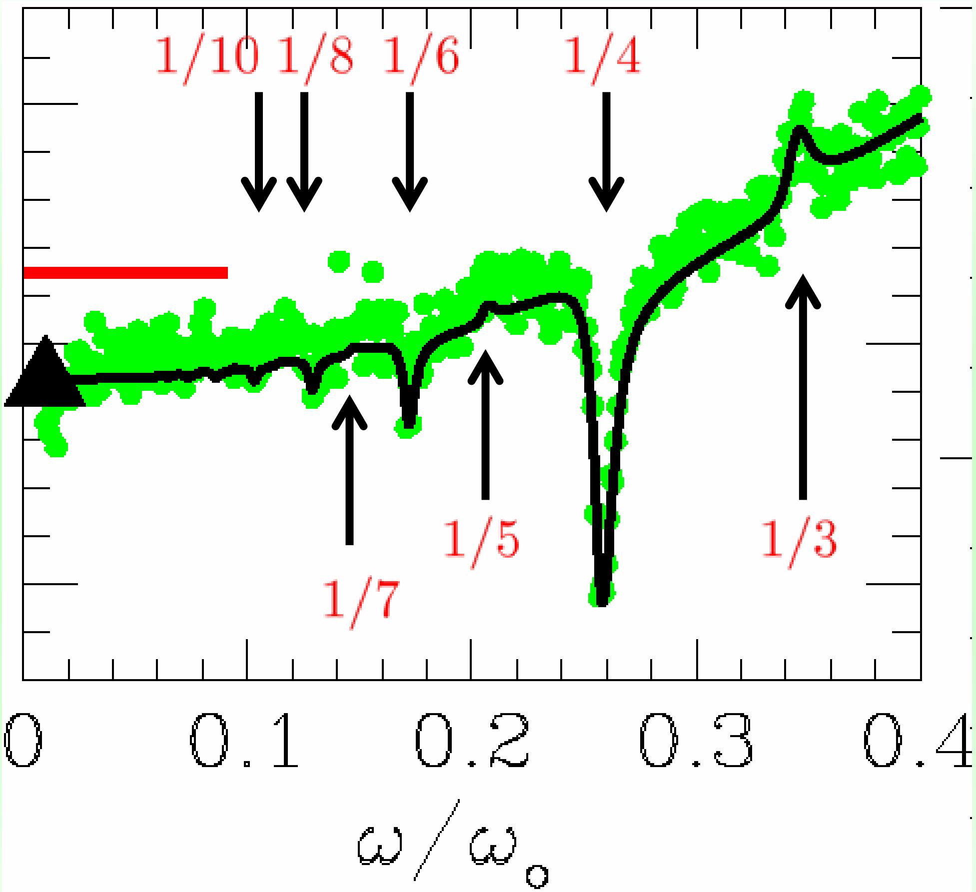
ω/ω_o=1.02

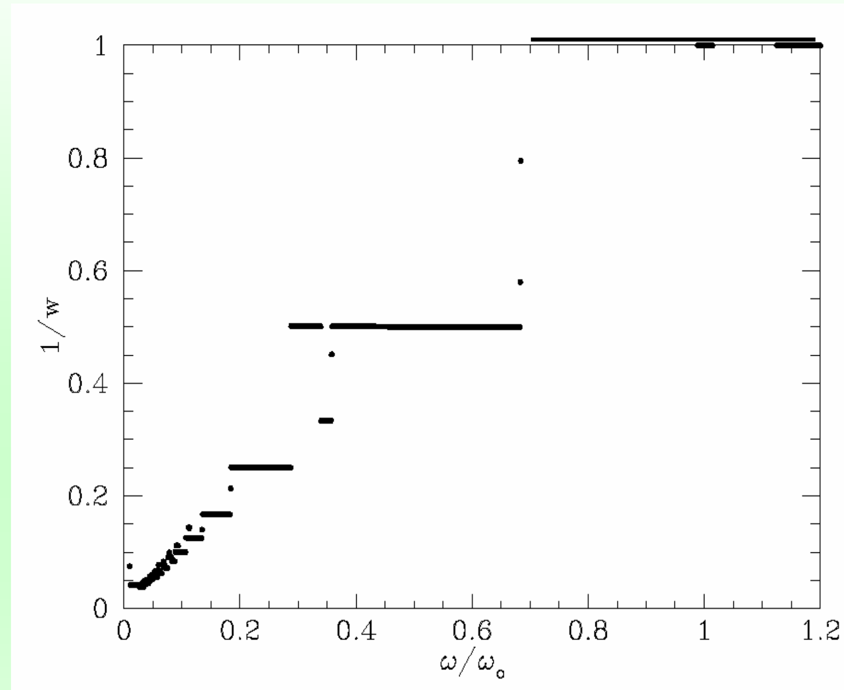
Negative cu ent

ω/ω_o=1.1

$\phi \approx \pi$

Devil's stai case of f equencies



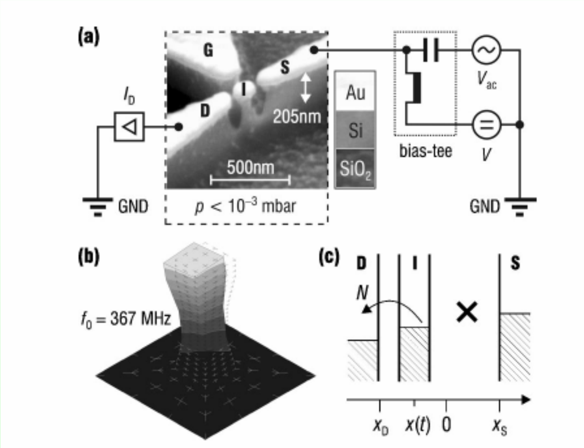


non-linea stochastic fo ce lead to f equency locking

fo $\omega = \omega_o/n$

Recent expe iment obse ved the main esonance

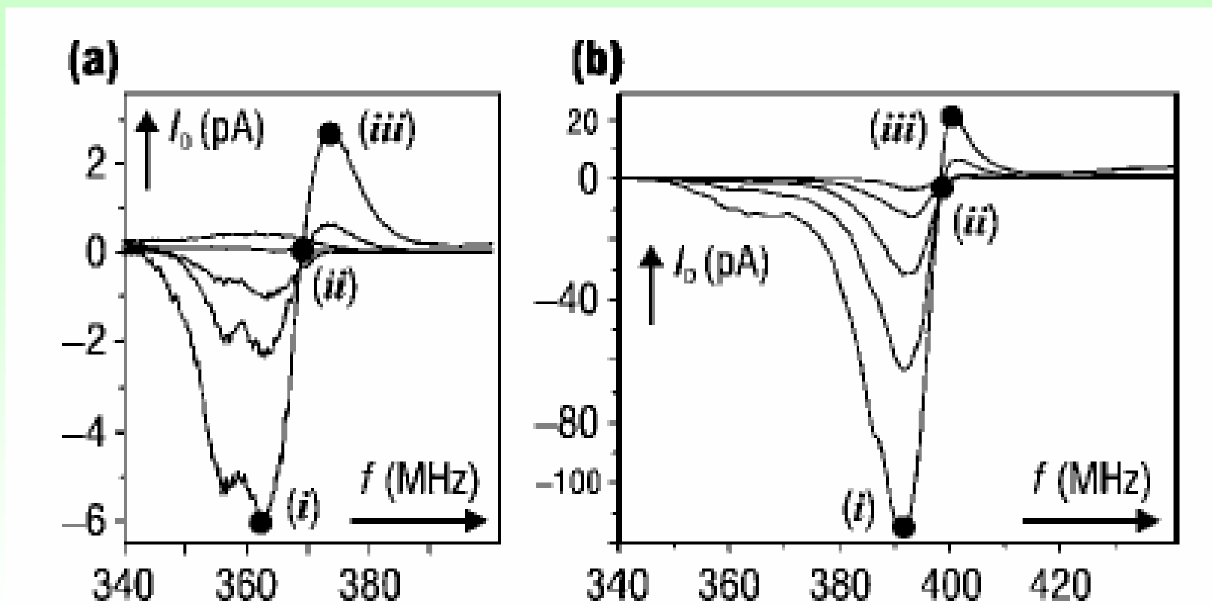
D. Sheible and R. Blick APL 2004



Not in Coulomb egime

Room tempe atu e and $E_C \approx$ 80 K

But main
featu e p esent



St uctu es at lowe

f equencies p esent

(p ivate communication)

Instability in presence of an AC drive

Fou ie expansion (without fo cing)

$$x(t)/\lambda = \sum_{\nu=1}^{\infty} [A_\nu \sin(\nu\omega t) + B_\nu \cos(\nu\omega t)] + B_0/2$$

$$\sum_n P_n(t) n = \sum_{\nu=1}^{\infty} [C_\nu \sin(\nu\omega t) + D_\nu \cos(\nu\omega t)] + D_0/2$$

$$\begin{pmatrix} A_\nu \\ B_\nu \end{pmatrix} = \frac{\epsilon}{(\frac{\nu^2\omega^2}{\omega_o^2} - 1)^2 + (\frac{\eta\nu\omega}{\omega_o})^2} \begin{pmatrix} 1 - \nu^2\omega^2/\omega_o^2 & \eta\nu\omega/\omega_o \\ -\eta\nu\omega/\omega_o & 1 - \nu^2\omega^2/\omega_o^2 \end{pmatrix} \begin{pmatrix} C_\nu \\ D_\nu \end{pmatrix}$$

At esonance:

$$\omega_R^2/\omega_o^2 \approx 1 - \eta C_1/D_1$$

Resonance shift

$$\eta\, A_1 = \epsilon D_1(A_1)$$

Ene gy balance

Fo small A_1/λ: analytical exp essions

$$C_1/\lambda = -a\frac{2\tilde{\Gamma}}{1+4\tilde{\Gamma}^2}\left[1+\frac{a^2}{4}\frac{1-4\tilde{\Gamma}^2}{1+4\tilde{\Gamma}^2}\right] \qquad D_1/\lambda = a\frac{\tilde{\Gamma}}{1+4\tilde{\Gamma}^2}\left[1+\frac{a^2}{4}\frac{1-12\tilde{\Gamma}^2}{1+4\tilde{\Gamma}^2}\right]$$

Instability fo

$$\epsilon > \epsilon_c = \eta(1+4\tilde{\Gamma}^2)/\tilde{\Gamma}$$

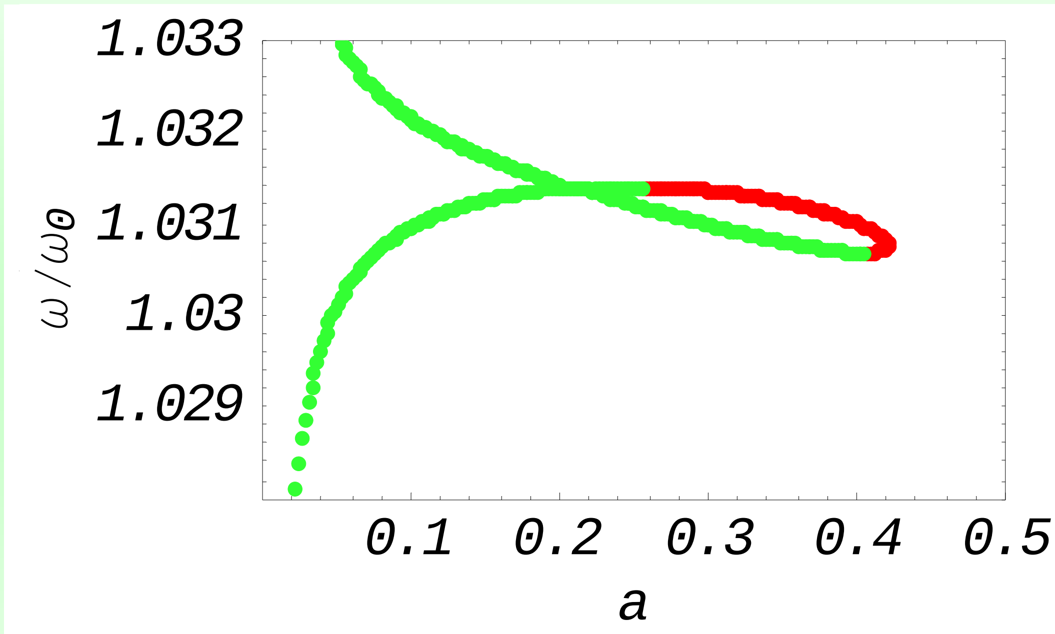
In p esence of the AC d ive:

$$\epsilon \rightarrow \epsilon[1+\alpha\sin(\omega t+\delta)] \qquad P_n(t) \rightarrow P_n(t)[1+\alpha\sin(\omega t+\delta)]$$

Slow motion of $\delta(t)$ and $a(t)=A_1(t)/\lambda$

$$\frac{d}{d\omega t}\begin{pmatrix} a \\ \delta \end{pmatrix} \approx \begin{pmatrix} 1/2 & \eta/4 \\ -\eta/4a & 1/2 \end{pmatrix}\cdot\begin{pmatrix} -\eta a+\epsilon D_1/\lambda \\ a(\omega_0^2/\omega^2-1)+\epsilon C_1/\lambda \end{pmatrix}$$

Evolution of the stability



Stationa y UNSTABLE points

Stationa y STABLE points

$$\tilde{\Gamma} = 1, \; \eta = 0.03, \; \alpha = 0.002$$

F.P. and R. Fa io, New Jou nal of Physics (2006)

Full counting statistics

F. P. Phys. Rev. B (2004)

F.P. and R. Fa io, New Jou nal of Physics (2006)

T anspo t desc ibed by a maste equation

one can obtain the FCS in the following way:

Bag ets and Na a ov PRB 2003

$$e^{-S_{t_o}(\chi)} = \langle q|\mathrm{Texp}\left\{-\int_0^t \hat{L}_\chi(t')dt'\right\}|p(0)\rangle \qquad |q\rangle \equiv \{1,1\}$$

$$\hat{L}_\chi(t) = \begin{pmatrix} \Gamma_L(t) & -\Gamma_R(t) \\ -\Gamma_L(t)e^{i\chi} & \Gamma_R(t) \end{pmatrix}$$

We assume **perfect periodic** motion x(t)=λ a sin(ω t)

$$e^{-S_N(\chi)} = \langle q|\hat{A}^N|p(0)\rangle \qquad \hat{A} = \mathrm{Texp}\left\{-\int_0^{2\pi} \hat{L}_\chi(\phi')d\phi'\right\}$$

Fo N→ ∞ the la gest (in modulus) eigenvalue of A su vives $\lambda_M(\chi)$

$$S_N(\chi) = -N\ln[\lambda_M(\chi)]$$

Full Counting Statistics (analytic)

$$e^{-S_N/N} = \frac{e^{-4\tau}}{2}[(1-2\alpha)(y^2-1)+2y\sinh(4\tau y) + (1+2\alpha+(1-2\alpha)y^2)\cosh(4\tau y)]$$

$$\tau \approx \Gamma/a$$

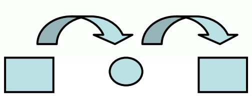
T inomial dist ibution fo $\tau \ll 1$

$$e^{-S_N(\chi)} = [\beta_0 + \beta_1 e^{i\chi} + \beta_2 e^{2i\chi}]^N$$

$$\begin{cases} \beta_0 = 2\alpha(1-4\tau+8\tau^2) \\ \beta_1 = 1-2\alpha(1-4\tau+4\tau^2)-4\tau^2 \\ \beta_2 = 4\tau^2(1-2\alpha) \quad . \end{cases}$$

Maximum p obability of t ansfe ing 1 elect on

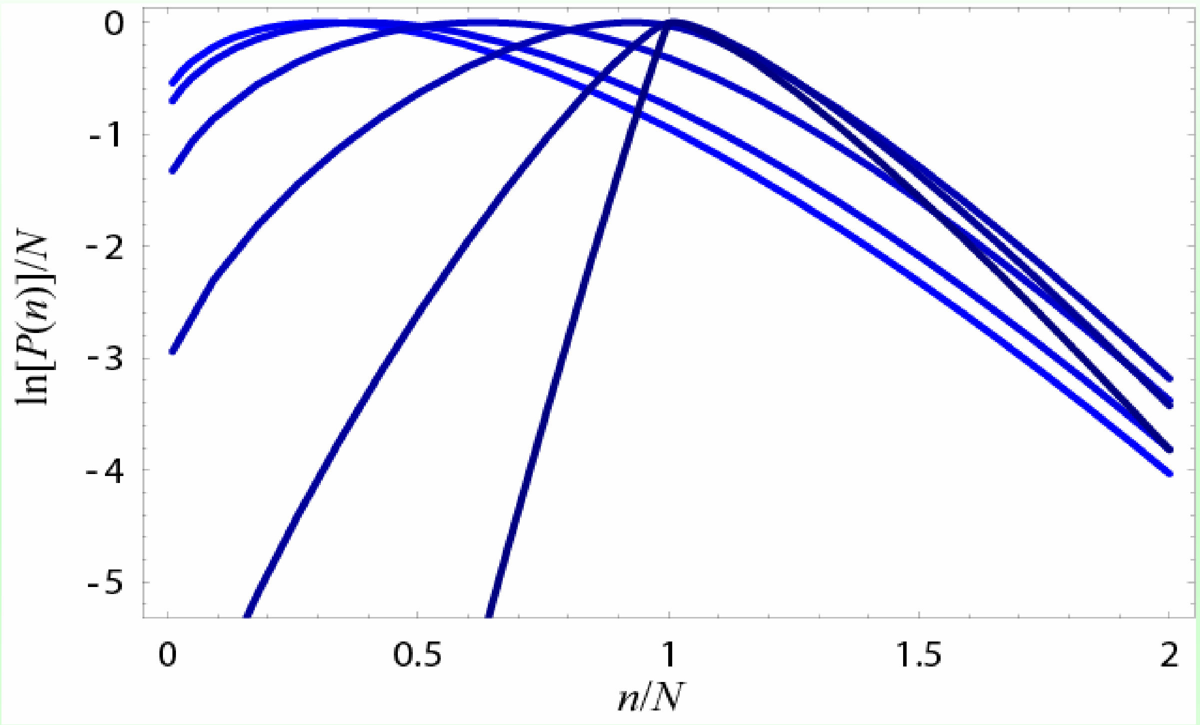
P obability of no t ansfe at all cont olled by α



P obability of t ansfe of 2 elect ons cont olled by τ

Nume ical esults:

Full counting statistics for Γ=0.1



F.P. Phys. Rev. B (2004)

Related pape s

Full Counting Statistics of Cooper Pair Shuttling
A. Romito and Yu. V. Na a ov, Phys. Rev. B. 2004
Full counting statistics of nano-electromechanical systems
C. Flindt, T. Novotny, and A.-P. Jauho, Eu oPhys. Lett. 2005

Full Counting Statistics for the AC forced shuttle

The same app oach can be used

In the adiabatic limit simple analytical esults

$$\mathcal{S}_N(\chi)/N = \int_0^{2\pi/\omega} \lambda_1(\chi,t)dt$$

Fi st fou cumulants:

$$C_1 = \gamma\epsilon\frac{\pi}{4}\beta(1-\beta^2)$$

$$C_2 = \gamma(1-\beta^2)\left[(1+\beta^2) - \epsilon\frac{\pi}{8}\beta^2(1-3\beta^2)\right]$$

$$C_3 = \gamma\epsilon\frac{\pi}{16}\beta(1-\beta^2)(1-12\beta^2+15\beta^4)$$

$$C_4 = \gamma\frac{(1-\beta^2)}{4}\left[(1+\beta^2-9\beta^4+15\beta^6)\right.$$

$$\left.-\epsilon\frac{\pi}{8}\beta^2(1-39\beta^2+135\beta^4-105\beta^6)\right]$$

Adiabatic pa amete

$$\gamma = eV_o\Gamma/(4E_C\omega) \gg 1$$

Asymmet y pa amete

$$\beta = \frac{\Gamma_L(0)-\Gamma_R(0)}{\Gamma_L(0)+\Gamma_R(0)}$$

Conclusions

-The cha ge shuttle fo ced by an AC voltage shows a ectification effect

- The non-linea ity is due to the stochastic occupation of the g ain
  Int insic not dependent on the mechanical esponse

-The development of the instability in p esence of an AC field lead to st ong non-linea ities in the esponse that can be studied expe imentally

- Cu ent fluctuations give insight on the dynamics of the device

Pe spective: Quantum AC fo ced shuttle

In this di ection: quantum noise in AC biased chaotic cavities:

D. Bag ets and F. Pistolesi, cond-mat/June06